Physics SN3

Problem Set #7: Nuclear Physics

Solutions

A01. C

A02. C

A03. C

A04. C

A05. A

A06. D

A07. B

A08. Fusion of elements lighter than iron and fission of nuclei heavier than iron are both exothermic (they release energy) because in both cases, the resulting nuclei have a higher binding energy than the reactants.

A09. D

A10. E

A11. A

A12. E

A13. C

A14. E

A15. Protons repel each other via the electric force (given by Coulomb's Law). When close enough, protons and neutrons attract each other via the nuclear strong force. Thus, adding neutrons in the nucleus increases the attraction via the strong force without adding repulsion via the electric force. (This effect is limited by the shot range of the strong force.) The graph shows that as Z increases, N increases more significantly.

Bonus question 1: There are gaps in the Segrè plot because some numbers of protons, and some numbers of neutrons are not stable. Example 43 protons, (technetium) is not stable and 21 neutrons is not stable either.

Bonus question 2: When N or Z is close to 20, 28, 50, 82 (or N=126) there are a lot of stable elements (these are so special that they called *magic* numbers). These correspond to full energy levels *within* the nucleus, and thus to an especially stable structure. Conversely, there are cases where a lonely proton (or neutron) by itself in the highest energy level is not a stable configuration, and the atom eventually beta-decays into a more stable isotope element (notice on the graph that there are no stable elements with N=19 or N=21 neutrons).

A16. C

A17. C

A18. D

B01.

a) The atomic mass (m) is the weighted average of the isotope masses:
$m = (m_1 \times abundance_1) + (m_2 \times abundance_2)$

Convert percentages to decimals:
m = (62.9298 × 0.6915) + (64.9278 × 0.3085)
m = 43.5066 + 20.0295 = 63.5361 u

Answer: The atomic mass of the element is 63.5 u

b) Isotope 1:
Nucleons = mass number = A = 63 (round atomic mass to nearest integer)
Protons = atomic number = Z = 29 (given in the problem)
Neutrons = N = A - Z = 63 - 29 = 34

Isotope 2:
Nucleons = mass number = A = 65 (round atomic mass to nearest integer)
Protons = atomic number = Z = 29 (given in the problem)
Neutrons = N = A - Z = 65 - 29 = 36

B02.

a) $r = r_0 A^{\frac{1}{3}} = (1.2 \text{ fm}) \left(238^{\frac{1}{3}}\right) = 7.44 \text{ fm}$

b) The volume V of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

Substitute $r = 7.44$ fm :

$$V = \frac{4}{3}\pi(7.44)^3 = \frac{4}{3} \times 3.1416 \times 412.3 = 1728.7 \text{ fm}^3$$

Density is mass per unit of volume $\rho = \frac{m}{V}$, where m is the mass of the uranium-238 nucleus (238 u), and $1\text{u} = 1.66054 \times 10^{-27}$ kg :

$$m = 238 \times 1.66054 \times 10^{-27} \text{ kg} = 3.9521 \times 10^{-25} \text{ kg}$$

Now calculate the density:

$$\rho = \frac{3.9521 \times 10^{-25} \text{ kg}}{1.7287 \times 10^{-42} \text{ m}^3} \approx 2.29 \times 10^{17} \text{ kg/m}^3$$

Answer: The density of the uranium-238 nucleus is approximately 2.29×10^{17} kg/m^3.

B03. a) Equation 10.4 from our OpenStax textbook is:

$$\Delta m = Zm_p + (A - Z)m_n - m_{nuc}$$

Since we are given atomic masses and not nuclear masses and data tables commonly include atomic masses (not nuclear masses), we will modify our equation slightly.

$$\Delta m = Zm_H + (A - Z)m_n - m_{atom}$$

We use the hydrogen mass, so we have Z = 6 extra electrons (obviously not part of the nucleus) but then those 6 extra electrons are also part of the atom, so they get removed at the end of the operation. This approximation may be too crude for nuclear physicists, but at this stage of your education, it is sufficient.

Where:

- $Z = 6$ (number of protons)
- $A = 12$ (mass number)

$$\Delta m = (6 \times 1.007\,825) + (6 \times 1.008\,665) - 12 = 0.098\,940 \text{ u}$$

b) $E = \Delta mc^2 = \left(0.098940u \cdot 931.494 \frac{MeV/c^2}{u}\right) \cdot c^2 = 92.2MeV$

c) $BEN = \frac{E_b}{A} = \frac{92.2MeV}{12} = 7.68\ \frac{MeV}{nucleon}$

B04. First, find the number of Carbon-14 atoms in 1 kg :

$$N_{\text{C}} = \frac{1000 \text{ g}}{12.0 \text{ g/mol}} \times 6.022 \times 10^{23} \text{ atoms /mol} = 5.02 \times 10^{25} \text{ atoms}$$

The number of Carbon-14 atoms:

$$N_{C-14} = (5.02 \times 10^{25}) \times (1.3 \times 10^{-12}) = 6.52 \times 10^{13} \text{ atoms}$$

Now calculate the decay constant:

$$\lambda = \frac{\ln 2}{5730} = 1.21 \times 10^{-4} \text{yr}^{-1}$$

Activity:

$$A = \lambda N_{C-14} = (1.21 \times 10^{-4}) \times (6.52 \times 10^{13}) = 7.89 \times 10^{9} \text{ decays / year}$$

Converting to decays per second (Bq) using 1 year = $3.16 \times 10^7 s$:

$$A = \frac{7.89 \times 10^9}{3.16 \times 10^7} = 250 \text{ Bq}$$

B05. If 80% of the ^{14}C in the sample has decayed, 20% of the ^{14}C remains.
Use the radioactive decay equation:

$$N(t) = N_0 e^{-\lambda t}$$

Since $N(t)/N_0 = 0.20$, we can write:

$$0.20 = e^{-\lambda t}$$

Taking the natural logarithm of both sides:

$$\ln(0.20) = -\lambda t$$
$$t = \frac{\ln(0.20)}{-\lambda}$$

The decay constant is:

$$\lambda = \frac{0.693}{5730} = 1.21 \times 10^{-4} \text{yr}^{-1}$$

Substituting into the equation for t :

$$t = \frac{\ln(0.20)}{-1.21 \times 10^{-4}} = \frac{-1.6094}{-1.21 \times 10^{-4}} = 13{,}300 \text{ years}$$

B06.

a) Number of atoms in 1 g of U: $N = \frac{1g \times 6.02 \cdot 10^{23} \frac{atoms}{mole}}{238.0 \frac{g}{mole\ U}} = 2.53 \times 10^{21}$ atoms U

Nb of atoms of $^{238}_{92}U$: $N_{U-238} = 0.9928 \times 2.53 \cdot 10^{21} = 2.51 \cdot 10^{21}$ atoms

Nb of atoms of $^{235}_{92}U$: $N_{U-235} = 0.0072 \times 2.53 \cdot 10^{21} = 1.82 \cdot 10^{19}$ atoms

$$A = -\frac{dN}{dt} = -\frac{d\left(N_o e^{-\lambda t}\right)}{dt} = -\left(-\lambda N_o e^{-\lambda t}\right) = \lambda N$$

$$\lambda_{U-238} = \frac{ln2}{4.47 \cdot 10^9 yr} = 1.55 \cdot 10^{-10} yr^{-1}$$

$$\lambda_{U-235} = \frac{ln2}{7.04 \cdot 10^8 yr} = 9.84 \cdot 10^{-10} yr^{-1}$$

$$A = \lambda_{U-238} N_{U-238} + \lambda_{U-235} N_{U-235} = 4.07 \cdot 10^{11} yr^{-1}$$

b) The table does not list the abundance of the shorter-lived (and thus more active) isotopes of uranium. I is reasonable to assume that a sample of uranium would contain some of these shorter-lived isotopes. Thus, our estimate is an under-estimate.

B07.

a) $^{239}_{94}Pu \rightarrow ^{235}_{92}U + ^{4}_{2}He + (Q\ or\ energy)$

OR

$^{239}_{94}Pu \rightarrow ^{235}_{92}U + \alpha + (Q\ or\ energy)$

b) The mass difference Δm is:

$$\Delta m = m(\,^{239}\text{Pu}) - [m(\,^{235}\text{U}) + m(\,^{4}\text{He})]$$

$$\Delta m = 239.052157 - (235.043924 + 4.002602) = 0.005631\text{ u}$$

Using $1\text{u} = 931.494\text{ MeV}/c^2$, the energy released is:

$$E = \Delta m \times 931.494 = 0.005631 \times 931.494 = 5.23\text{ MeV}$$

c) The energy carried by the alpha particle is:

$$E_\alpha = 0.80 \times 5.23 = 4.18\text{ MeV}$$

B08.

a) From the N vs Z thorium-232 decay series, we count 6 alpha decays and 4 beta decays.

Only alpha decays affect the mass number, so let's start by counting how many it takes to go from the starting mass number of 232 to the final mass number of 208, which was also done in a previous problem.

Each alpha decay decreases the mass number by 4. To go from 232 to 208, the total change in mass number is:

$$232 - 208 = 24$$

Since each alpha decay reduces the mass number by 4, the number of alpha decays is:

$$\frac{24}{4} = 6\text{ alpha decays}$$

Thus, there are 6 alpha decays.

Now let's analyze the atomic number change from 90 to 82.

Each alpha decay decreases the atomic number by 2, and each beta decay increases the atomic number by 1.

The change in atomic number from the 6 alpha decays previously found is:

$$6 \times (-2) = -12$$

So, after 6 alpha decays, the atomic number would be:

$$90 - 12 = 78$$

But we must reach a final atomic number of 82, so the number of beta decays needed (since each increases the atomic number by 1) must be:

$$82 - 78 = 4 \text{ beta decays}$$

Therefore, the total number of alpha and beta decays is consistent with what we found on the diagram.

b) By logic after 1 half-life 50% remains; after 2 half-lives 50% of 50%, and therefore 25% remains: t = 2.80×10^{10} years

OR, we calculate:

For 25% remaining $(N(t)/N_0 = 0.25)$: $N(t) = N_0 e^{-\lambda t} \rightarrow \frac{1}{4} = e^{-\lambda t}$

$$\ln\left(\frac{1}{4}\right) = -\lambda t \rightarrow ln(4) = \lambda t \rightarrow t = \frac{ln4}{\lambda} \rightarrow t = \frac{ln4}{\frac{ln2}{T_{1/2}}}$$

$$t = \frac{ln4}{\frac{ln2}{T_{1/2}}} = \frac{ln4}{ln2} T_{1/2} = 2 \cdot T_{\frac{1}{2}} = 2 \cdot 1.40 \times 10^{10} = 2.80 \times 10^{10} \; years$$

B09.

a) The atomic number goes up by 1 and the mass number stays the same, this means that a neutron turned into a proton. The decay illustrated is a β^-.
The diagram then shows energy levels and wavelengths corresponding to downward arrows between energy levels. This illustrates a gamma decay: the nucleus was in an excited state, and decays to its ground state by emitting a gamma ray.

The equations are:

$$^{99}_{42}Mo \rightarrow {}^{99}_{43}Tc * + e^- + \overline{\upsilon}$$

$$^{99}_{43}Tc * \rightarrow {}^{99}_{43}Tc + \gamma$$

b) $E_{initial} = E_{final} + E_{\gamma}$

$E_{\gamma} = E_{initial} - E_{final} = 0.922 - 0.140 = 0.782 \; keV$

$$E_\gamma = hf = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E_\gamma} = \frac{4.14 \cdot 10^{-15} \times 3 \cdot 10^8}{0.782 \cdot 10^3} = 1.59 \cdot 10^{-9} m$$

B10.

Equation of the fusion: ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n + (Q\ or\ energy!)$

$$\Delta m = (2 \cdot m({}^{2}_{1}H) - m({}^{3}_{2}He) - {}^{1}_{0}n)$$

$$\Delta m = (2 \cdot 2.014\ 102 - 3.016\ 029 - 1.008\ 665) = 0.003510\ u$$

$$E = \Delta m c^2 = \left(0.003510u \cdot 931.494 \frac{MeV/c^2}{u}\right) c^2 = 3.27\ MeV$$