

Problem Set #5: Diffraction
Solutions

Part A

Question 1

D

Question 2

C

Question 3

A

Question 4

E

Question 5

C

Question 6

A

Question 7

B

Question 8

$\lambda_{\text{sound}} > \lambda_{\text{visible light}}$ so there is a much greater degree of diffraction

Question 9

The long axis should be vertical so that the sound is diffracted more horizontally (throughout the concert hall). Just like for single-slit diffraction of light, where a horizontal slit would diffract the light on the screen in a vertical pattern, and a vertical slit would diffract the light on the screen in a horizontal pattern. This argument is only valid for relatively high frequencies overall.

Question 10

$d \sin \theta = m\lambda$ As the number of slits increases, the primary maxima get narrower

Part B

Question 1

$$\lambda = 550 \text{ nm}$$

$$R = 2.50 \text{ m}$$

$$y = 3.5 \text{ mm because } (7 \text{ mm}) / 2$$

$a \sin \theta = m\lambda$ is the formula for diffraction minima

$$\sin \theta = \frac{y}{R}$$

$$a \frac{y}{R} = m\lambda$$

$m = 1$ because we are dealing with the location of the first diffraction minima

$$a = \frac{\lambda R}{y} = \frac{550 \times 10^{-9} \times 2.5}{3.5 \times 10^{-3}} = 3.93 \times 10^{-4} \text{ m}$$

Question 2

$$\lambda = 630 \text{ nm}$$

$$a = 0.15 \text{ mm}$$

$$R = 3.00 \text{ m}$$

a)

$a \sin \theta = m\lambda$ for diffraction minima

$$\sin \theta = \frac{y}{R}$$

$$a \frac{y}{R} = m\lambda$$

$m = 1$ because we wish to find the position of the first minimum

$$y = \frac{m\lambda R}{a} = \frac{1 \times 630 \times 10^{-9} \times 3.00}{0.15 \times 10^{-3}} = 1.26 \times 10^{-2} \text{ m}$$

So the first diffraction minima is 1.26 cm from the center of the pattern

b)

the point halfway between the center of the pattern and the first intensity minimum is

$$\frac{1.26 \times 10^{-2}}{2} = 6.3 \times 10^{-3} \text{ m}$$

NOTE: This solution uses the old textbook's definition of beta. You will get the same final answer if you use the OpenStax textbook's version of beta.

$$\beta = \frac{2\pi}{\lambda} a \sin \theta \quad \text{with} \quad \sin \theta = \frac{y}{R}$$

We find $\beta = \pi$

$$I = I_0 \left(\frac{2}{\pi} \sin \frac{\pi}{2} \right)^2$$

$$I = 0.405 I_0$$

$$I = 40.5\% I_0$$

The intensity halfway to the first minimum is 40.5% of the intensity at the center.

Question 3

$$R = 4.00 \text{ m}$$

$$\lambda = 546 \times 10^{-9} \text{ m}$$

From the picture, the first interference maximum is at 1 cm and the interference maximum at 4 cm is missing.

a)

$$\sin \theta = \frac{y}{R}$$

$$d \sin \theta = m\lambda \text{ for interference maximum}$$

$$m = 1 \text{ for the first minimum}$$

$$d \frac{y}{R} = m\lambda$$

$$d = \frac{\lambda R}{y} = \frac{546 \times 10^{-9} \times 4.00}{1 \times 10^{-2}} = 2.18 \times 10^{-4} \text{ m}$$

b)

$$a \sin \theta = m\lambda \text{ for diffraction minima}$$

$$a \frac{y}{R} = m\lambda$$

$$a = \frac{m\lambda R}{y} = \frac{1 \times 5.46 \times 10^{-7} \times 4}{4 \times 10^{-2}} = 5.46 \times 10^{-5} \text{ m}$$

The width of the slit is 54.6 μm

Question 4

$$\lambda = 630 \text{ nm}$$

$$R = 4.00 \text{ m}$$

$$\text{Central maximum to first minimum in single-slit diffraction } y = 3.15 \times 10^{-2} \text{ m}$$

$$\text{The distance between two successive interference maxima } y = 1.05 \times 10^{-2} \text{ m}$$

a)

$$a \sin \theta = m\lambda \text{ for diffraction minimum}$$

$$m = 1 \text{ for first minimum}$$

$$\sin \theta = \frac{y}{R}$$

$$a = \frac{m\lambda R}{y} = \frac{630 \times 10^{-9} \times 4.00}{3.15 \times 10^{-2}} = 8.00 \times 10^{-5} \text{ m}$$

The width of the slits is $80\text{ }\mu\text{m}$

b)

$d \sin \theta = m\lambda$ for interference maximum

$m = 1$ for the first interference maximum

$$\sin \theta = \frac{y}{R}$$

$$d \frac{y}{R} = m\lambda$$

$$d = \frac{m\lambda R}{y} = \frac{6.3 \times 10^{-7} \times 4}{1.05 \times 10^{-2}} = 2.40 \times 10^{-4} \text{ m}$$

The distance between the slits is 0.24 mm

c)

$a \sin \theta = 1\lambda$ first diffraction minimum

$d \sin \theta = m\lambda$ m^{th} interference maximum at same position ($\sin \theta$ is the same for the diffraction minimum and the interference maximum)

$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{1\lambda}$$

$$m = \frac{d}{a} = \frac{2.40 \times 10^{-4}}{8.00 \times 10^{-5}} = 3$$

So the first missing maximum is 3.

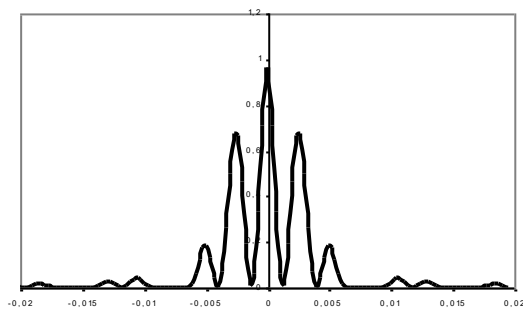
d)

We have the central maximum (that's 1), then we have two maxima on each side (a total of 4) before the third on each side is missing.

So the total number of interference maxima in the central maxima 5.

e)

Intensity from the double-slit as a function of the angle from the center of the slits to the screen



Question 5

Diffraction grating 350 lines per mm $\frac{350}{10^{-3}} = 350000$ lines per m

$$d = \frac{1}{350000} = 2.83 \times 10^{-6} \text{ m}$$

Visible spectrum is 400-700nm.

a)

We must find the angular width of the first order i.e. $m=1$

$$d \sin \theta = m\lambda$$

$$\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right)$$

$$\text{For 400 nm : } \theta = \sin^{-1} \left(\frac{1 \times 400 \times 10^{-9}}{2.86 \times 10^{-6}} \right) = 8.04^\circ$$

$$\text{For 700 nm: } \theta = \sin^{-1} \left(\frac{1 \times 700 \times 10^{-9}}{2.86 \times 10^{-6}} \right) = 14.17^\circ$$

$$\text{Angular width: } 14.17^\circ - 8.04^\circ = 6.13^\circ$$

The angular width of the first order is 6.13°

b)

We must find the angular width of the 3rd order i.e. $m=3$

$$\text{For 400 nm : } \theta = \sin^{-1} \left(\frac{3 \times 400 \times 10^{-9}}{2.86 \times 10^{-6}} \right) = 24.8^\circ$$

$$\text{For 700 nm: } \theta = \sin^{-1} \left(\frac{3 \times 700 \times 10^{-9}}{2.86 \times 10^{-6}} \right) = 47.2^\circ$$

$$\text{Angular width: } 47.2^\circ - 24.8^\circ = 22.4^\circ$$

The angular width of the first order is 22.4°

Question 6

This is an interference problem from reflected rays, which obeys the same principle as interference through slits.

$$d \sin \theta = m\lambda$$

$$\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right)$$

$$m = 1 \quad \theta = \sin^{-1} \left(\frac{1 \times 632.8 \times 10^{-9}}{1.6 \times 10^{-6}} \right) = 23.3^\circ$$

$$m = 2 \quad \theta = \sin^{-1} \left(\frac{2 \times 632.8 \times 10^{-9}}{1.6 \times 10^{-6}} \right) = 52.3^\circ$$

$m=3$ does not exist

the intensity of the reflected light will be maximum at 0° , 23.3° , and 52.3° .

Question 7

Using Malus's Law:

$$\begin{aligned}\frac{I}{I_0} &= \cos^2 \theta \\ 0.100 &= \cos^2 \theta \\ \theta &= \cos^{-1}(\sqrt{0.100}) = 71.6^\circ\end{aligned}$$

Question 8

After the first polarizing filter, the intensity of unpolarized light is reduced by 50.0%. Then, after the second polarizing filter, using Malus's law:

$$I_2 = (0.500 \cdot I_0) \cdot \cos^2 30.0^\circ = 37.5 \% I_0$$

Question 9

After the first polarizing filter, the intensity of unpolarized light is reduced by 50.0%. Then, after the second polarizing filter, using Malus's law:

$$I_2 = (10.0 \text{ W/cm}^2) \cdot \cos^2(62.0 - 25.0) = 6.38 \text{ W/cm}^2$$

Question 10

(a) The beam is unpolarized and passes through a polarizing filter, making its irradiance decrease by 50.0%. So, the beam will leave that filter with an irradiance of 500 W/m².

(b) The orientation is found using Malus's law:

$$\begin{aligned}319 &= 500 \cos^2 \theta \\ \theta &= \cos^{-1}(\sqrt{319/500}) = 37.0^\circ\end{aligned}$$