Physics SN3

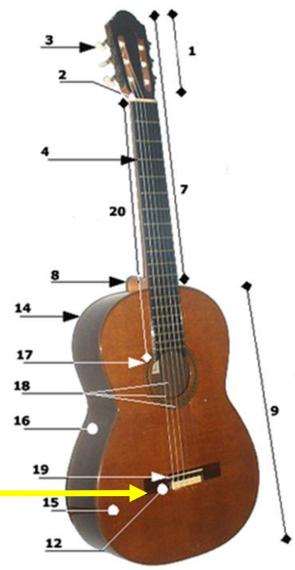
Problem Set #3: Sound

Solutions

Part A

1. D
2. E
3. D
4. B
5. A
6. C
7. E
8. A
9. D
10. B
11. an empty bottle is an open-closed tube with fundamental frequency f=v/4L. Filling the bottle with liquid makes the ‘tube’ shorter and makes the fundamental frequency higher.

12. the string attached to the guitar makes the louder sound, because it’s not the actual string that makes the sound: the string is very small and can’t move much air. The string makes the bridge vibrate, which makes the top plate of the instrument vibrate. The top plate of the instrument is what moves the air, creating sound. (picture from Wikipedia)
Reference: https://www.phys.unsw.edu.au/music/guitar/



Part B

B-01. a) The intensity is proportional the power of the source and inversely proportional to the distance square:

$$I=\frac{\langle P\rangle}{A}=\frac{\langle P\rangle}{4\pi r^2}=\frac{2\times 10^{-3}\,\mathrm{W}}{4\pi(8\,\mathrm{m})^2}=2.48\times 10^{-6}\,\mathrm{W/m^2}$$

b) The intensity (I) and the intensity level (β) are two distinct variables.

$$\beta(\mathrm{dB})=10\log\left(\frac{I}{I_o}\right)=10\log\left(\frac{2.48\times10^{-6}\,\mathrm{W/m^2}}{1\times10^{-12}\,\mathrm{W/m^2}}\right)=63.95dB\approx 64dB$$

c) we cannot simply add *log*, therefore, we have to find the intensity of the 4 dogs at 8m and then find the intensity level.

$$I_{\mathrm{tot}}=4I=4\times(2.48\times10^{-6}\,\mathrm{W/m^2})=9.92\times10^{-6}\,\mathrm{W/m^2}$$

$$\beta(\mathrm{dB})=10\log\left(\frac{I}{I_o}\right)=10\log\left(\frac{9.92\times10^{-6}\,\mathrm{W/m^2}}{1\times10^{-12}\,\mathrm{W/m^2}}\right)=69.97\,\mathrm{dB}\approx 70\,\mathrm{dB}$$

B-02.

a) We can find the intensity level from the intensity:

$$I = \frac{\langle P \rangle}{A} = \frac{P}{4\pi r^2} = \frac{20\,\mathrm{W}}{4\pi(3\mathrm{m})^2} = 0.177\ \mathrm{W/m^2}$$

$$\beta(\mathrm{dB}) = 10\log\left(\frac{I}{I_o}\right) = 10\log\left(\frac{0.176\,\mathrm{W/m^2}}{1\times10^{-12}\,\mathrm{W/m^2}}\right) = 112\,\mathrm{dB}$$

b) from the sound intensity equation, we can isolate to find the intensity

$$\beta(\mathrm{dB}) = 10\log\left(\frac{I}{I_o}\right) \Rightarrow I = I_o \times 10^{\left(\frac{\beta(\mathrm{dB})}{10}\right)}$$

$$I = 10^{-12}\,\mathrm{W/m^2} \times 10^{\left(\frac{50}{10}\right)} = 1\times10^{-7}\,\mathrm{W/m^2}$$

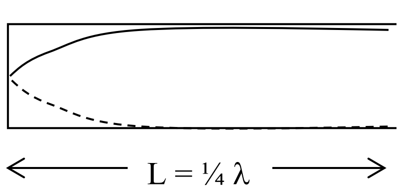
The power didn't change because the source is the same, therefore, we can find the distance at which the source is located:

$$I = \frac{\langle P \rangle}{A} = \frac{\langle P \rangle}{4\pi r^2} \square\ r = \sqrt{\frac{\langle P \rangle}{4\pi I}} = \sqrt{\frac{20\,\mathrm{W}}{4\pi(10^{-7}\,\mathrm{W/m^2})}} = 3.99\square10^3\,\mathrm{m} \square 4\,\mathrm{km}$$

c) Given that the intensity is $I = 0.176\,\mathrm{W/m^2}$ and the distance is $r = 3.99\square10^3 m$, we can find the power needed from the source

$$I = \frac{\langle P \rangle}{A} = \frac{\langle P \rangle}{4\pi r^2} \square\ \langle P \rangle = 4\pi r^2 I = 4\pi(3.98\square10^3\,\mathrm{m})^2(0.176\,\mathrm{W/m^2}) = 3.50\square10^7\,\mathrm{W}$$

3. a) The minimum length of pipe will corresponds to the length of the fundamental frequency

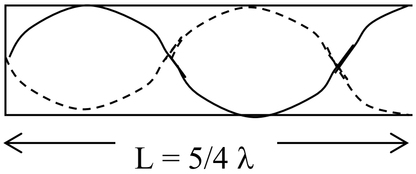


assume speed of sound $v = 343\,\mathrm{m/s}$

$$v = f\lambda \square\ \lambda = \frac{v}{f} = \frac{343\,\mathrm{m/s}}{300\,\mathrm{Hz}} = 1.14\,\mathrm{m}$$

$$L = \frac{1}{4}\lambda = \frac{1}{4}(1.14\,\mathrm{m}) = 0.286\,\mathrm{m}$$

b) For the fifth harmonic

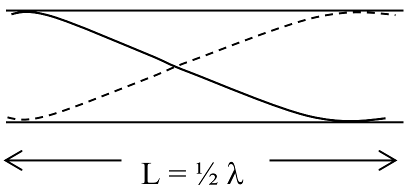


$$v = f\lambda \Rightarrow \lambda = 1.14m$$

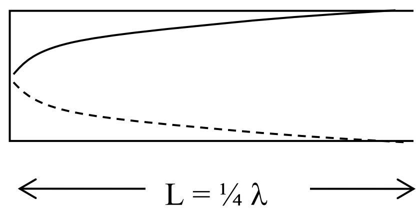
$$L = \frac{5}{4}\lambda = \frac{5}{4}(1.13\,\text{m}) = 1.43\,\text{m}$$

4.

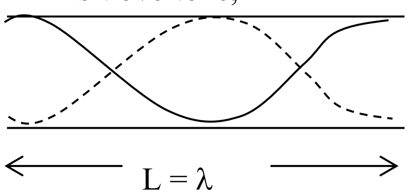
fundamental



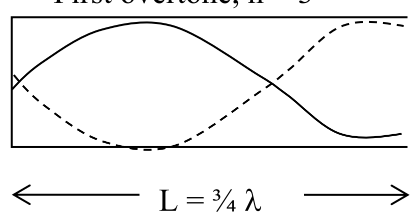
fundamental



Fisrt overtone, n = 2



First overtone, n = 3



Given that v = 343 m/s

$$v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{300\,\text{Hz}} = 1.14\,\text{m}$$

$$L_{\text{open}} = \frac{\lambda}{2} = \frac{1.14}{2} = 0.572\,\text{m}$$

and the frequencies are equal for the first overtone of the closed and open pipe.

$$f_{\text{closed}} = f_{\text{open}} = 600\,\text{Hz}$$

$$\frac{3v}{4L_{\text{closed}}} = \frac{2v}{2L_{\text{open}}}$$

$$L_{\text{closed}} = \frac{3\left(L_{\text{open}}\right)}{4} = \frac{3(0.572\,\text{m})}{4} = 0.429\,\text{m}$$

[Make sure that you understand the signification of *overtone* and *harmonic*, the possible wavelengths and frequencies associated with the open and closed pipe.]

5.

a) From the given information, we have no way to be sure if the tension in the guitar string is too high or too low. All we know is that the guitar string's frequency differs from the tuning fork by 4 Hz.

$$f_{\text{beat}} = \left| f_{\text{guitar}} - f_{\text{fork}} \right| = 4\,\text{Hz} \Rightarrow f_{\text{guitar}} = 294\,\text{Hz} \pm 4\,\text{Hz}$$

We know that

$$|v| = \lambda f_{\text{guitar}} = \sqrt{\frac{F_{\text{T}}}{\mu}} \Rightarrow f_{\text{guitar}} = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{F_{\text{T}}}{\mu}}$$

The wavelength λ of the fundamental is fixed by the guitar's length and the mass density μ doesn't change much — the string's tension is primarily what's responsible for fixing the fundamental frequency.

How can she know if her guitar string is sharp (frequency too high) or flat (frequency too low)? By slightly increasing the tension and seeing its effect of the beat frequency. If f_{guitar} is already too high, increasing the tension will increase its difference from the tuning fork and therefore increasing f_{beat}; if it's too low, f_{beat} will decrease when the tension is increased.

If $f_{\text{guitar}} < f_{\text{fork}}$:

$$f_{\text{guitar}} = 290\,\text{Hz}$$

$$290\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{\text{actual}}}{\mu}}$$

$$294\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{\text{ideal}}}{\mu}}$$

$$\frac{294\,\text{Hz}}{290\,\text{Hz}} = \sqrt{\frac{F_{\text{ideal}}}{F_{\text{actual}}}} \Rightarrow F_{\text{ideal}} = \left(\frac{294}{290}\right)^2 F_{\text{actual}}$$

$$F_{\text{ideal}} = 1.028 F_{\text{actual}}$$

If $f_{\text{guitar}} > f_{\text{fork}}$:

$$f_{\text{guitar}} = 298\,\text{Hz}$$

$$298\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{\text{actual}}}{\mu}}$$

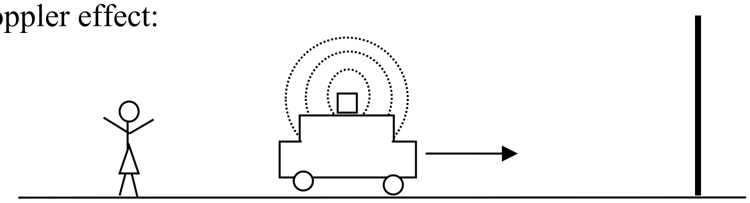
$$294\,\text{Hz} = \frac{1}{\lambda}\sqrt{\frac{F_{\text{ideal}}}{\mu}}$$

$$\frac{294\,\text{Hz}}{298\,\text{Hz}} = \sqrt{\frac{F_{\text{ideal}}}{F_{\text{actual}}}} \Rightarrow F_{\text{ideal}} = \left(\frac{294}{298}\right)^2 F_{\text{actual}}$$

$$F_{\text{ideal}} = 0.973 F_{\text{actual}}$$

So, if the guitar is tuned to low, its tension needs to be increased by 2.8%. If it's tuned to high, the tension needs to be reduced by 2.7%.

6. Using Doppler effect:



a) the listener is at rest and the siren moves away from the listener:

$$f_{\text{listener}} = f_{\text{o}} = f_{\text{s}}\left(\frac{v}{v+v_{\text{s}}}\right) = f_{\text{siren}}\left(\frac{v}{v+v_{\text{siren}}}\right) = (1000\,\text{Hz})\left[\frac{343\,\text{m/s}}{(343+10)\,\text{m/s}}\right] = 971.7\,\text{Hz}$$

b) the cliff (acting as the observer) is at rest and the siren moves toward the cliff

$$f_{\text{cliff}} = f_{\text{o}} = f_{\text{s}}\left(\frac{v}{v-v_{\text{s}}}\right) = f_{\text{siren}}\left(\frac{v}{v-v_{\text{siren}}}\right) = (1000\,\text{Hz})\left[\frac{343\,\text{m/s}}{(343-10)\,\text{m/s}}\right] = 1030.0\,\text{Hz}$$

The sound reflects off of the cliff, with the cliff acting as an emitter of a 1030.0 Hz signal, heard by the listener. As both listener (observer) and the cliff (source) are the rest,

$$f'_{\text{listener}} = f_{\text{cliff}}\left(\frac{v}{v}\right) = 1030.0\,\text{Hz}$$

c) the beat frequency: $f_{\text{beat}} = \left|f'_{\text{listener}} - f_{\text{listener}}\right| = \left|(1030.0\,\text{Hz}) - (971.7\,\text{Hz})\right| = 58.3\,\text{Hz}$

d) repeat the same procedure as in parts a, b and c but now the listener is moving toward the siren at 5m/s.

$$f_{\text{listener}} = f_{\text{siren}}\left(\frac{v+v_{\text{listener}}}{v+v_{\text{siren}}}\right) = (1000\,\text{Hz})\left[\frac{(343+5)\,\text{m/s}}{(343+10)\,\text{m/s}}\right] = 985.8\,\text{Hz}$$

$$f_{\text{cliff}} = f_{\text{siren}}\left(\frac{v}{v-v_{\text{siren}}}\right) = (1000\,\text{Hz})\left[\frac{343\,\text{m/s}}{(343-10)\,\text{m/s}}\right] = 1030.0\,\text{Hz}$$

$$f'_{\text{listener}} = f_{\text{cliff}}\left(\frac{v+v_{\text{listener}}}{v}\right) = (1030.0\,\text{Hz})\left[\frac{(343+5)\,\text{m/s}}{343\,\text{m/s}}\right] = 1045.0\,\text{Hz}$$

$$f_{\text{beat}} = \left|f'_{\text{listener}} - f_{\text{listener}}\right| = \left|(1045\,\text{Hz}) - (985.8\,\text{Hz})\right| = 59.2\,\text{Hz}$$

7. Using Doppler effect:

S ←+ L
Sub 1 Sub 2
$v_s = -6 m/s$ $v_L = 29 m/s$

a) Both subs are moving toward each other

$$f_{sub_2} = f_{sub_1}\left(\frac{v+v_{sub_2}}{v-v_{sub_1}}\right) = (2000\,\text{Hz})\left[\frac{(1540+29)\,\text{m/s}}{(1540-6)\,\text{m/s}}\right] = 2045.6\,\text{Hz}$$

Sub 2 detects a 2045.6 Hz signal.

b) This 2045.6 Hz signal is reflected from sub 2; i.e., sub 2 now acts as an emitter of a 2045.6 Hz signal. To find the frequency of the reflected signal detected by sub 1, we compute a second Doppler shift, this time with sub 2 acting as the source and sub 1 as the observer.

$$f'_{sub_1} = f_{sub_2}\left(\frac{v+v_{sub_1}}{v-v_{sub_2}}\right) = (2045.6\,\text{Hz})\left[\frac{(1540+6)\,\text{m/s}}{(1540-29)\,\text{m/s}}\right] = 2093\,\text{Hz}$$