Physics SN3

Problem Set #2: Travelling Waves and Standing Waves

Solutions

Part A

1. C
2. A
3. A
4. $v = \frac{\lambda}{T} = \frac{4m}{0.5s} = 8m/s$ A
5. C
6. B
7. C
8. E
9. Longer length, same speed
10. a) bigger T (tension); higher speed
    b) bigger μ; lower speed

Part B

1. a) The wavelength of a wave depends on the frequency and the speed

$$v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{345}{262} = 1.32m$$

b) A phase change of 90° corresponds to one-quarter of a period.

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{2(262)} = 9.54\times10^{-4}s$$

c) We know that $1\lambda \leftrightarrow 2\pi rad$

$$\frac{6.4cm}{132cm} = \frac{\Delta\phi}{2\pi} \Rightarrow \Delta\phi = \frac{2\pi(6.4)}{132} = 0.305rad$$

d) To find the equation of the wave, we need the amplitude, the wave number, the angular frequency and the phase.

$$A = 0.02cm$$

$$k = \frac{2\pi}{\lambda} = 4.76rad/m$$

$$\omega = 2\pi f = 1650rad/s$$

at t = 0 and x = 0; phase $\frac{3\pi}{2}$ (for sine wave…. What would it be for cosine wave? Try it!)

The horizontal displacement from equilibrium position:

$$s(x,t) = 0.02\sin(4.76x \mp 1650t + \frac{3\pi}{2})$$

Note that the sign is – is the wave travels to the right and + is the wave travels to the left.

2. a) According to the wave equation, the amplitude is $0.001m$

b) the wavelength depends on the wave number

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{62.8} = 0.100m$$

c) $k = 62.8 rad/m$

d) the frequency is related to the angular frequency:

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50.0Hz$$

e) The speed does not depend on the frequency nor the wavelength:

$$v = f\lambda = 50 \times 0.1 = 5.00m/s$$

$$\text{or} \quad v = \frac{\omega}{k} = 5.00m/s$$

f) left

g) The transverse velocity and the wave speed are two distinct things; make sure that you know the difference.

$$v_y = \frac{dy}{dt} = 314 \times (0.001)\cos(62.8x + 314t)$$

$$v_y = 314 \times (0.001)\cos(62.8(0.05) + 314(1.2)) = -0.308 \text{ m/s}$$

3. According to the standing wave equation:

a) The amplitude of the standing wave: $2A = 0.50cm$
Therefore, the amplitude of the individual wave: $0.25cm$

The speed can be obtained from the wave number and angular frequency

$$v = \frac{\omega}{k} = \frac{500}{0.025} = 2\times10^4 cm/s = 200 m/s$$

b) The frequency is related to the angular frequency:

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = 79.6 Hz$$

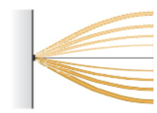
The wavelength is related to the wavenumber:

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = 251 cm$$

c) Internode distance always corresponds to $\frac{1}{2}\lambda$ = 126 cm

d) We have one end fixed (thus a node) and one end free (thus an antinode).

The shortest scenario that fits this looks like a quarter wavelength thus: $\frac{126}{2} = 63 cm$



e) The maximum displacement occurs when $\cos(\omega t) = \cos(500t) = 1$

$$A_x = [0.5\sin(kx)]$$

$$A_{0.30} = 0.5\sin(0.025\times30) = 0.341 cm$$

4. a) The speed depends only on the force and density:

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{16}{2.5\times10^{-2}}} = 25.3 m/s$$

We can find the period and frequency from the speed equation and from the graph where the wavelength is found:

$$v = f\lambda = \frac{\lambda}{T} \Rightarrow T = \frac{\lambda}{v} = \frac{4}{25.3} = 0.158 s$$

$$f = \frac{1}{T} = 6.32 Hz$$

b) We can find the transverse velocity from the wave equation

$$y = A\sin(kx - \omega t)$$

$$v_y = \frac{dy}{dt} = -\omega A\cos(kx - \omega t)$$

$$v_{y_{\max}} = \omega A = 2\pi fA = 2\pi(6.32)(2) = 79.4cm/s$$

c) The wave equation is found given the wave number, the angular frequency, the phase and the amplitude.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = \frac{\pi}{2} rad/m \qquad \omega = 2\pi f = 39.7 rad/s$$

$$A = 0.02m \qquad \phi = 0$$

the wave travels to the right, therefore, we use the – sign:

$$y = A\sin(kx - \omega t)$$

$$y = 0.02\sin\left(\frac{\pi}{2}x - 39.7t\right)$$

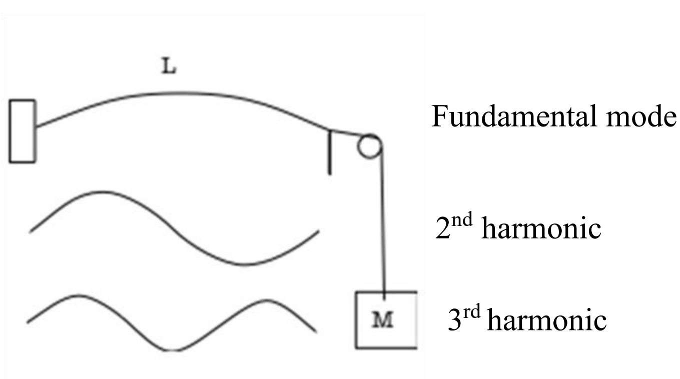
d) at t = 0, and x = 0, the wave is ¾ the way through a cosine function. So the phase is $\phi = -\frac{\pi}{2} = \frac{3\pi}{2}$. Therefore, the wave function is also:

$$y = 0.02\cos\left(\frac{\pi}{2}x - 39.7t + \frac{3\pi}{2}\right)$$

e) The average power is giving by:

$$P = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(0.025)(39.7)^2(0.02)^2(25.3) = 0.199W$$

5.



a) In the fundamental mode: $L = \frac{1}{2}\lambda$

$$\lambda = 3.0m \qquad f = 60Hz$$

$$v = f\lambda = 180m/s$$

$$v = \sqrt{\frac{F_T}{\mu}} \Rightarrow F_T = v^2\mu = (180)^2(8\times10^{-4}) = 25.9N$$

b) 2nd harmonic: $L = \lambda$

$$\lambda = 1.5m$$

$$v = f\lambda = 90m/s$$

$$F_T = v^2\mu = (90)^2(8\times10^{-4}) = 6.48N$$

More directly: velocity halved $\rightarrow$ F_T reduced to ¼ original

c) 3rd harmonic: $L = \frac{3}{2}\lambda$

$$\lambda = 1m$$

$$v = f\lambda = 60m/s$$

$$F_T = v^2\mu = (60)^2(8\times10^{-4}) = 2.88N$$

F_T is 1/9 F_T fundamental

6. a) The speed $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{720}{5\times10^{-2}}} = 120m/s$

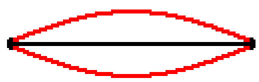
b) $\mathbf{n}f_0 = 280$ Hz and $(\mathbf{n+1})f_0 = 350$

Therefore, $f_0 = 70\text{Hz}$

280Hz corresponds to n = 4
350Hz corresponds to n = 5

c) 70 Hz (found in b))

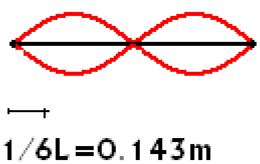
d)



$$v = f\lambda \Rightarrow \lambda = \frac{v}{f} = \frac{120}{70} = 1.71m$$

$$L = \frac{1}{2}\lambda = \frac{1}{2}(1.71m) = 0.857m$$

e)



$$y = A\sin kx \underbrace{\cos\omega t}_{\max=1}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.857} = 7.33 rad/m$$

$$y_{\max} = 4\sin(7.33\times 0.143) = 3.47cm$$