Supplementary Problems 3: Magnetism

1. An electron has a velocity $\boldsymbol{v} = (3.00\boldsymbol{\hat{\imath}} - 2.00\boldsymbol{\hat{\jmath}}) \times 10^5 \frac{m}{s}$ when it enters a region where there is a uniform magnetic field $\boldsymbol{B} = \left(-1.20\boldsymbol{\hat{\imath}} + 4.00\boldsymbol{\hat{k}}\right)T$. What force does the electron initially experience?

$$\begin{aligned}\vec{F} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} &= (-1.60 \times 10^{-19})\big((3.00\boldsymbol{\hat{\imath}} - 2.00\boldsymbol{\hat{\jmath}}) \times 10^5\big) \times \left(-1.20\boldsymbol{\hat{\imath}} + 4.00\boldsymbol{\hat{k}}\right) \\ &= (-1.60 \times 10^{-19})(10^5)\begin{vmatrix} \boldsymbol{\hat{\imath}} & \boldsymbol{\hat{\jmath}} & \boldsymbol{\hat{k}} \\ 3 & -2 & 0 \\ -1.2 & 0 & 4 \end{vmatrix} \\ &= (-1.60 \times 10^{-14})\left(\boldsymbol{\hat{\imath}}(-8) - \boldsymbol{\hat{\jmath}}(12) + \boldsymbol{\hat{k}}(-2.4)\right) \\ &= \left(1.28\hat{\imath} + 1.92\hat{\jmath} + 0.38\hat{k}\right) \times 10^{-13}\ \text{N}\end{aligned}$$

2. A singly charged ^{24}Mg ion (mass = 24u) is accelerated from rest by a potential difference of 2.00×10^3 V and then bent in a uniform magnetic field of 5.00×10^{-2} T in a mass spectrometer. Find the radius of curvature of the orbit of the ion. Include a diagram showing all vectors.

   This is a classical mass spectrometer question. We start by using conservation of energy to find out the speed of the particle as it exits the acceleration plates:

$$\frac{1}{2}mv^2 = q\Delta V \rightarrow v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19})(2.00 \times 10^3)}{(24 \times 1.67 \times 10^{-27})}} = 1.26 \times 10^5\ \text{m/s}$$

   Once we have the speed, it is a simple formula to find the radius of curvature of the orbit

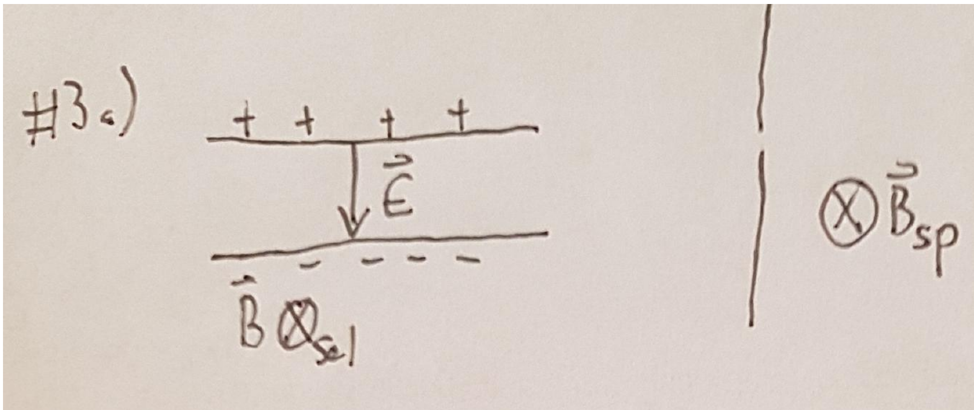
$$r = \frac{mv}{qB} = \frac{(24 \times 1.67 \times 10^{-27})(1.26 \times 10^5)}{(1.60 \times 10^{-19})(5.00 \times 10^{-2})} = 0.631\ \text{m}$$

3. A mass spectrograph is preceded by a velocity selector consisting of a set of parallel plates separated by a distance of 2.00 mm and having a potential difference of 1.60×10^2V. The magnetic induction in the selector is 0.420 T, and the induction in the spectrograph is 1.20 T. Find:

   (a) The speed with which the ions enter the spectrograph. Include a diagram showing the relative orientation of all vectors.

   A velocity selector only allows charges through undeflected if they have the desired speed by applying electric and magnetic forces in opposite directions thanks to electric and magnetic fields perpendicular to the path of the charges (see diagram below). In order for these forces to cancel out, they must have equal magnitude.

$$|qE| = |qvB\sin 90°| \rightarrow v = \frac{E}{B} = \frac{\Delta V/d}{B} = \frac{(1.60 \times 10^2)}{(4.20)(2.00 \times 10^{-3})} = 1.90 \times 10^5\ \text{m/s}$$



(b) The separation distance of the peaks on a photographic plate for singly charged ^{24}Mg and ^{26}Mg ions.

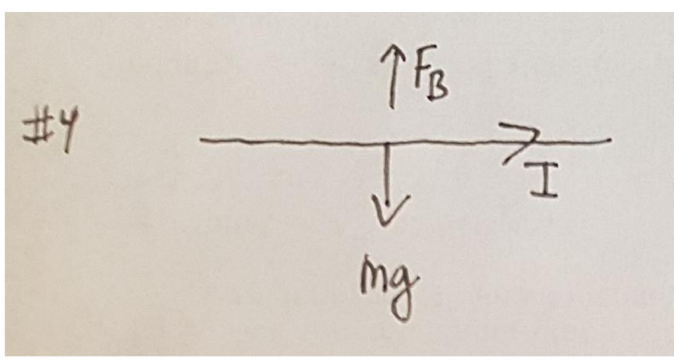
We find the radius of curvature of the two different ions with $r = \frac{mv}{qB}$

$$r_{24} = \frac{(24 \times 1.67 \times 10^{-27})(1.90 \times 10^5)}{(1.60 \times 10^{-19})(1.20)} = 3.97 \text{ cm}$$

$$r_{26} = \frac{(26 \times 1.67 \times 10^{-27})(1.90 \times 10^5)}{(1.60 \times 10^{-19})(1.20)} = 4.30 \text{ cm}$$

The ions travel a semi-circle in the magnetic field, so they hit the detector a distance of diameter=2radius away. The difference in distance is thus $2r_{26} - 2r_{24} = 0.33$ cm

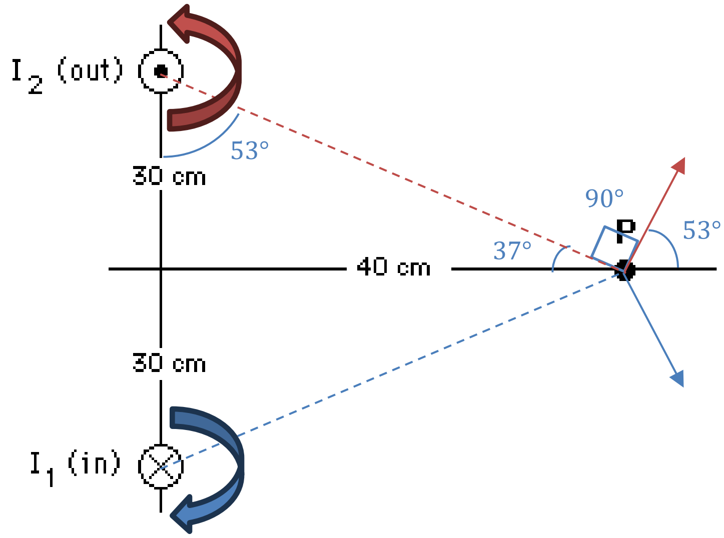
4. A horizontal current carrying wire 0.200 m long is mounted on a balance. Perpendicular to the wire is a uniform horizontal magnetic field of magnitude 0.500 T. The downwards force on the wire exerted by the magnetic field is measured by the balance and is found to be 0.240 N. What is the magnitude of the current in the wire?



The magnetic force on the wire has magnitude $F = |I\vec{l} \times \vec{B}| = IlB \sin 90° = IlB$. The current is thus

$$I = \frac{F}{lB} = \frac{0.240}{0.200 \cdot 0.500} = 2.40 \text{ A}$$

5. The figure below is an end view of two long, straight, parallel wires each carrying a current of 20A in the direction indicated.



(a) What is the resultant magnetic field at P due to the two wires?

The distance between the wire and point P is $R = \sqrt{0.30^2 + 0.40^2} = 0.50$ m.

Magnetic fields produced by straight wires are circular, directed according to the corkscrew right-hand-rule. For I_1, this is ccw and for I_2 this is cw. At point P, the magnetic field from each wire points tangent to these circles, at angles of $\theta = \pm 53°$

The magnitude of the magnetic field created by each wire is

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7})(20)}{(2\pi)(0.50)} = 8.0 \times 10^{-6}\ \mathrm{T}$$

In component form, according to the diagram above,

$$\vec{B}_1 = 8.0 \times 10^{-6}(\cos 53°\,\hat{\imath} - \sin 53°\,\hat{\jmath})\ \mathrm{T}$$

$$\vec{B}_2 = 8.0 \times 10^{-6}(\cos 53°\,\hat{\imath} + \sin 53°\,\hat{\jmath})\ \mathrm{T}$$

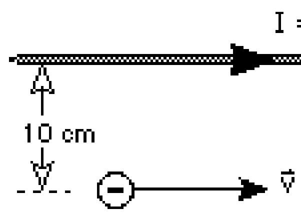
The x components add up while the y components cancel out, so the net magnetic field at P is

$$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 = 2 \times 8.0 \times 10^{-6} \times \cos 53°\,\hat{\imath} = 9.6 \times 10^{-6}\,\hat{\imath}\ \mathrm{T}$$

(b) What magnetic force would be exerted on a third wire, parallel to the first two, placed at P, with length 10cm, and carrying a current of 30A out of the page?

$$\vec{F} = I_3\vec{l} \times \vec{B}_{net} = (30)\big(0.100\,\hat{k}\big) \times (9.6 \times 10^{-6}\,\hat{\imath}) = 2.88 \times 10^{-5}\hat{\jmath}\ \mathrm{N}$$

6. A long straight wire carries a current of 1.50 A. An electron is moving with a speed 5.00 x 10^4m/s parallel to the wire in the same direction as the current. At the instant the electron is 10.0 cm from the wire, what magnetic force does it experience?



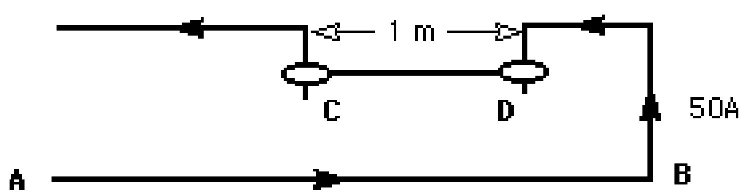
The magnetic field from the wire at the position of the electron points into the board and has magnitude

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7})(1.5)}{(2\pi)(0.10)} = 3.0 \times 10^{-6} T$$

The magnetic force on the electron is

$$\vec{F} = q\vec{v} \times \vec{B} = (-1.60 \times 10^{-19})(5.00 \times 10^{4}\hat{\imath}) \times \left(-3.0 \times 10^{-6}\,\hat{k}\right) = -2.40 \times 10^{20}\hat{\jmath}\text{ N}$$

7. A long horizontal wire, **AB**, rests on a horizontal surface. Another wire, **CD**, directly above the **AB**, and parallel to it, is 100cm long and is free to slide up and down on two metal guides **C** and **D**. The two wires are connected through the sliding contacts and carry a current of 50.0 A. If the mass of the wire **CD** is 5.00 grams, to what equilibrium height will it rise due to the magnetic force exerted on it by **AB**?

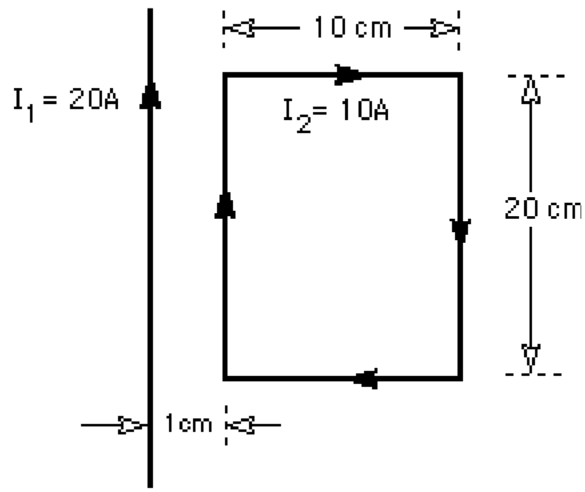


The wire is in equilibrium when the net force on it is zero, so when the magnitude of the magnetic force upward is equal to its weight downward.

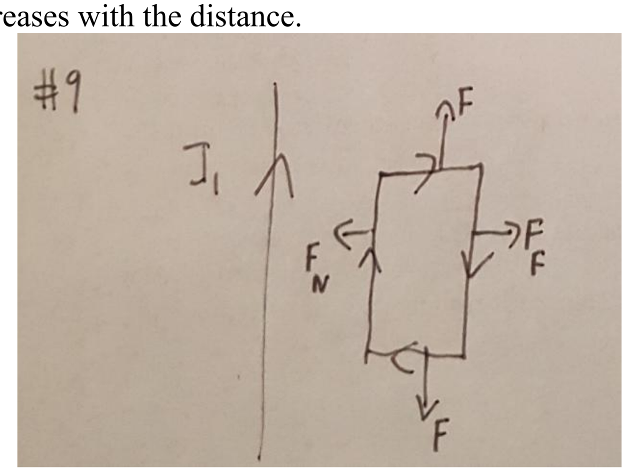
$$mg = IlB\sin 90^\circ = Il\frac{\mu_0 I}{2\pi R}$$

$$R = \frac{\mu_0 I^2 l}{2\pi mg} = \frac{(4\pi \times 10^{-7})(50.0)^2(1.00)}{(2\pi)(5.00 \times 10^{-3})(9.81)} = 1.02\text{ cm}$$

8. The long straight wire in the figure below carries a current of 20A. The rectangular loop, whose long sides are parallel to the wire, carries a current of 10A. Both the wire and loop are in the same plane. Determine the magnitude and direction of the resultant magnetic force exerted by the wire on the loop.



The net force on the loop is the sum of the force on each side, each given by $\vec{F} = I\vec{l} \times \vec{B}$. As shown below, the symmetry of the problem that the top and bottom segments' forces will cancel out. However, the near and far segment's forces do not cancel because the magnetic field created by the straight wire decreases with the distance.



Let's start by finding the magnitude of the force on the near segment.

$$F_N = I_2 l B \sin 90° = I_2 l \frac{\mu_0 I_1}{2\pi R_N} = (10)(0.20)\frac{(4\pi \times 10^{-7})(20)}{(2\pi)(0.01)} = 8.00 \times 10^{-4} \text{ N}$$

Likewise, the magnitude of the force on the far segment is

$$F_F = \frac{\mu_o I_1 I_2 l}{2\pi R_F} = \frac{(4\pi \times 10^{-7})(20)(10)(0.20)}{(2\pi)(0.01 + 0.10)} = 7.27 \times 10^{-5} N$$

Finally, the net force is

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_F = 8.00 \times 10^{-4}(-\hat{\imath}) + 7.27 \times 10^{-5}(\hat{\imath}) = -7.27 \times 10^{-4}\hat{\imath} \text{ N}$$

9. A closely wound coil has a diameter of 40.0 cm and carries a current of 2.50 A How many turns must it have if the magnetic field at its central axis is 1.26 x 10^{-4} T?

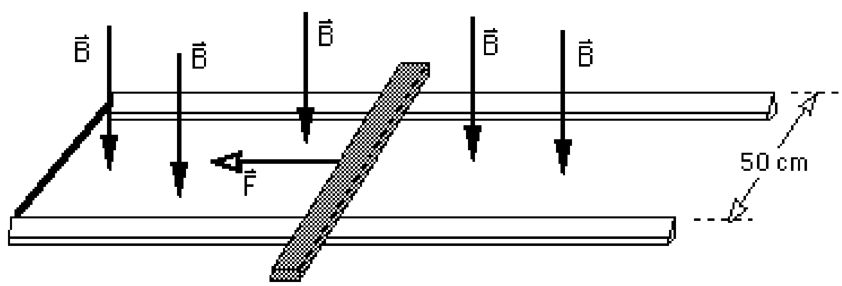
$$B_{coil} = \frac{\mu_0 NI}{2R}$$

$$N = \frac{2RB}{\mu_0 I} = \frac{2(0.200)(1.26 \times 10^{-4})}{(4\pi \times 10^{-7})(2.50)} = 16 \text{ turns}$$

10. A solenoid of length 20.0 cm and radius 2.00 cm is closely wound with 200 turns. If the current in the windings is 5.00 A, determine the magnetic field at the axis of the solenoid.

$$B_{sol} = \mu_0 nI = \mu_0 \frac{N}{L} I = 4\pi \times 10^{-7} \frac{200}{0.200} 5.00 = 6.28 \times 10^{-3}\ T$$

11. A 50.0 cm long bar with resistance 0.250 Ω is being pulled to the left along two horizontal, frictionless rails whose electrical resistance is zero. If the uniform magnetic field has a magnitude of 0.500 T in the direction shown, and the current in the rails is 1.50 A,



(a) what is the speed of the bar?

The induced EMF in the loop formed by the rails and the bar is given by the change in flux through the rectangular area:

$$|\mathcal{E}_{ind}| = N\left|\frac{d\Phi}{\mathrm{dt}}\right| = 1\frac{d}{dt}AB\cos 0° = B\frac{d}{dt}xy = By\frac{dx}{dt}$$

$$\rightarrow I_{ind}R = Blv$$

$$v = \frac{I_{ind}R}{By} = \frac{(1.50)(0.250)}{(0.500)(0.500)} = 1.50 \text{ m/s}$$

(b) What is the power expended by the pulling force ***F***?

Recalling mechanics, the instantaneous power expended by a force is $P = \vec{F} \cdot \vec{v} = Fv$ here since $\vec{F}$ and $\vec{v}$ are in the same direction. Since the bar is moving at constant speed, the net force on it must be zero, so $\vec{F}$ must be equal and opposite to the magnetic force on the bar due to the induced current flowing through it

$$F = F_m = I_{ind}lB\sin 90° = (1.50)(0.500)(0.500) = 0.375 \text{ N}$$

$$P = Fv = (0.375)(1.50) = 0.563 \text{ W}$$

(c) What is the rate of Joule heating in the bar?

The rate of Joule heating is the power dissipated by a current through a resistance

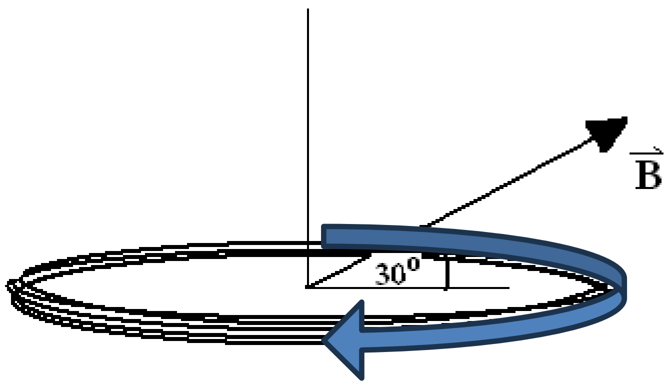
$$P = I_{ind}^2 R = (1.50)^2(0.250) = 0.563\ \mathrm{W}$$

As expected, this satisfies conservation of energy.

12. A 200 turn coil with area 0.15 m^2 is located in a uniform magnetic field which makes an angle of 30° with the plane of the coil. If the magnitude of the magnetic field changes with time according to the following equation

$$B = 0.05t + 0.01t^2$$

where B is in Teslas and t is seconds.



(a) What is the magnitude of the emf induced in the coil at t = 5s?

$\vec{A}$ is oriented normal to the area of the loop, so upward. The angle between $\vec{A}$ and $\vec{B}$ is thus $90 - 30 = 60°$

$$|\mathcal{E}_{ind}| = N\left|\frac{d\Phi}{\mathrm{dt}}\right| = N\frac{d}{dt}A\,B\cos(60°) = NA\cos(60°)\frac{dB}{dt}$$

$$= NA\cos(60°)\frac{d}{dt}(0.05t + 0.01t^2) = NA\cos(60°)\,(0.05 + 0.02t)$$

$$|\mathcal{E}_{ind}|(t = 5s) = (200)(0.15)\cos(60°)\,(0.05 + 0.02 \cdot 5) = 2.25\ \mathrm{V}$$

(b) What would then be the current in the coil if it had a total resistance of 1.5Ω?

$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = \frac{2.25}{1.5} = 1.5\ \mathrm{A}$$

(c) On the diagram above, indicated the direction of the induced current.

The magnetic field strength, and thus the magnetic flux through the loop, are increasing. This causes induction to oppose the change in flux, so an induced magnetic field pointing downward. The induced current is clockwise as seen from above