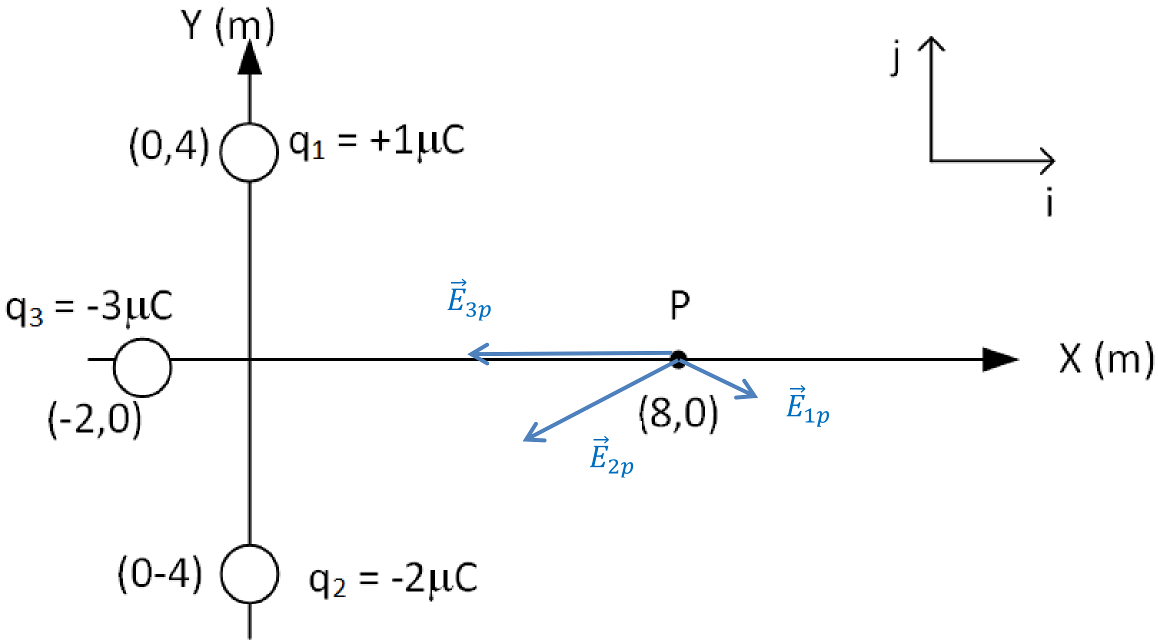
Supplementary Problems Solutions 1: Electrostatics

1. Consider the system of fixed point-charges shown in the figure below.



a) Determine the magnitude and direction of the electric field at the point P due to the point charges.

Let us first find the magnitude of the electric field at P from each charge and its direction, Then, we need to express each vector in component form to find the sum.

To do this, we must first find the distance from the charge to the point

$$r_{1p} = \sqrt{8^2 + 4^2} = 8.9\ m$$

The magnitude of the electric field of charge 1 at P is then

$$|\vec{E}_{1p}| = \left|\frac{kq_1}{r_{1p}^2}\right| = \frac{(8.99 \times 10^9)(1 \times 10^{-6})}{8.9^2} = 113.5\ N/C$$

Likewise,

$$r_{2p} = r_{1p}\sqrt{8^2 + 4^2} = 8.9\ m$$

$$|\vec{E}_{2p}| = \left|\frac{kq_2}{r_{2p}^2}\right| = \frac{(8.99 \times 10^9)(2 \times 10^{-6})}{8.9^2} = 227.0\ N/C$$

$$r_{3p} = 8 - (-2) = 10\ m$$

$$\left|\vec{E}_{3p}\right| = \left|\frac{kq_3}{r_{3p}^2}\right| = \frac{(8.99 \times 10^9)(3 \times 10^{-6})}{10^2} = 269.7\ N/C$$

Note we took the absolute value for each electric field here, so we have to be very careful with our directions. The electric field vectors point **away** from positive charges and **toward** negative charges.

The direction of $\vec{E}_1$ is down and to the right (quadrant 4) and the angle can be found from the position of the charge

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{4}{8}\right) = 26.6°$$

$$\theta_1 = -26.6°$$

$$\vec{E}_{1p} = 113.5\ N/C\ @ - 26.6°$$

Now, we need to express the electric field vector in component form

$$\begin{aligned}\vec{E}_{1p} &= 113.5\frac{N}{C}(\cos(-26.6°)\,\hat{\imath} + \sin(-26.6°)\,\hat{\jmath}) \\ &= (101.5\,\hat{\imath} - 50.8\,\hat{\jmath})N/C\end{aligned}$$

The direction of $\vec{E}_2$ is down and to the left (quadrant 3) and with the same angle found from the geometry

$$\theta_2 = 26.6° + 180° = 206.6°$$

$$\begin{aligned}\vec{E}_{2p} &= 227.0\ N/C\ @\ 206.6° \\ &= 227.0\frac{N}{C}(\cos(206.6°)\,\hat{\imath} + \sin(206.6°)\,\hat{\jmath}) \\ &= (-203.0\,\hat{\imath} - 101.6\,\hat{\jmath})N/C\end{aligned}$$

The direction of $\vec{E}_{3p}$ is to the left or at 180°, so in component form

$$\vec{E}_{3p} = -269.7\,\hat{\imath}\ N/C$$

Finally, we sum to find the net (total) electric field at point P.

$$\begin{aligned}\vec{E}_p &= \vec{E}_{1p} + \vec{E}_{2p} + \vec{E}_{3p} \\ &= (101.5\,\hat{\imath} - 50.8\,\hat{\jmath}) + (-203.0\,\hat{\imath} - 101.6\,\hat{\jmath}) + (-269.7\,\hat{\imath}) \\ &= (101.5 - 203.0 - 269.7)\hat{\imath} + (-50.8 - 101.6)\hat{\jmath} \\ &= (-371.2\,\hat{\imath}\ -\ 152.4\,\hat{\jmath})\ N/C\end{aligned}$$

In polar form,

$$E_{net} = \sqrt{E_x^2 + E_y^2} = \sqrt{(-371.2\,)^2 + (-\,152.4)^2} = 401\ N/C$$

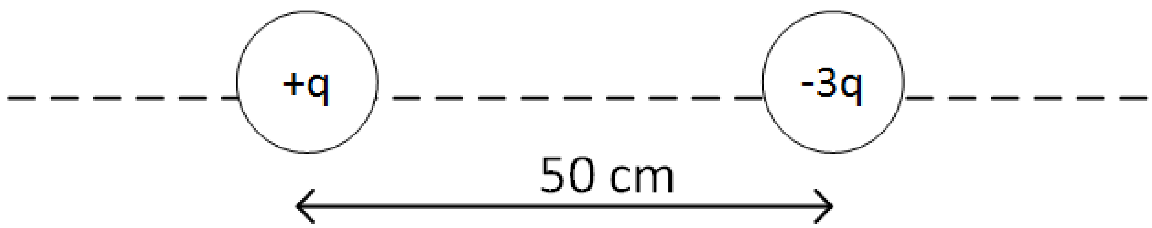
$$\theta = \arctan\left(\frac{E_y}{E_x}\right) = \arctan\left(\frac{-\,152.4}{-371.2}\right) + 180 = 21 + 180 = 201°$$

The net electric field at point P is 401 N/C at 201°

b) What force would a charge q = -4.00 × 10^{-8}C experience if placed at P?

$$\begin{aligned}\vec{F} &= q\vec{E}\\ &= (-4.00 \times 10^{-8})(401\ @201°)\\ &= 1.60 \times 10^{-5}N\ @\ (201 - 180) = 1.60 \times 10^{-5}N\ @21°\end{aligned}$$

2. In the figure below two fixed point charges, +q and -3q, are placed 50.0 cm apart in air.



a) Locate the finite point on the line passing through the two charges where the resultant electric field intensity is zero.

Let's place our origin at the positive charge (let's call is q_1) so its position is $x_1 = 0$ m and the negative charge (q_2)'s position is $x_2 = 0.50$ m.

For all positions $x < 0$ m, the electric field from q_1 is away from it, to the left, and the electric field from q_2 is toward it, to the right. Since q_2 has a larger magnitude than q_1 but is also a larger distance away, it is possible for the field from the two charges to cancel out

$$|E_1| = |E_2|$$

$$\left|\frac{kq_1}{r_1^2}\right| = \left|\frac{kq_2}{r_2^2}\right|$$

$$\frac{kq}{x^2} = \frac{k3q}{(0.50 - x)^2}$$

Cancelling out k and q, we obtain

$$3x^2 = (0.50 - x)^2 = x^2 - x + 0.25$$

$$\rightarrow 2x^2 + x - 0.25 = 0$$

Solving the quadratic equation, we find two solutions, $x = 0.183$ m and $x = -0.68$ m. Since this solution was derived for $x < 0$, we can only keep $x = -0.68$ m.

If we look at positions between the two charges, $0 < x < 0.50$ m, then the electric field from both charges is pointing to the right and there is no position where they cancel out.

If we look at position $x > 0.50$ m, the electric field from q_1 is away from it, to the right, and the electric field from q_2 is toward it, to the left. However, since q_2 had both a greater magnitude than q_1 and is closer, its contribution to the electric field is always stronger than q_1's and can never cancel out. The net electric field is always to the left and never 0.

Thus, the only position where the electric field is 0 is at $x = -0.68$ m, 68 cm to the left of the weaker positive charge.

b) Locate the finite point (or points) on the line where the electric potential is zero.

At all points in space, the potential is

$$V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \frac{kq}{|x|} + \frac{k(-3q)}{|0.50 - x|}$$

For positions $x < 0$ m, there can be 0 potential if

$$\frac{kq}{-x} = \frac{3kq}{0.50 + x}$$

Cancelling out k and q,

$$-3x = x + 0.50$$

$$x = -0.25 \text{ m}$$

For positions $0 < x < 0.50$ m, there can be 0 potential if

$$\frac{kq}{x} = \frac{3kq}{0.50 - x}$$

Cancelling out k and q,

$$3x = 0.50 - x$$

$$x = 0.125 \text{ m}$$

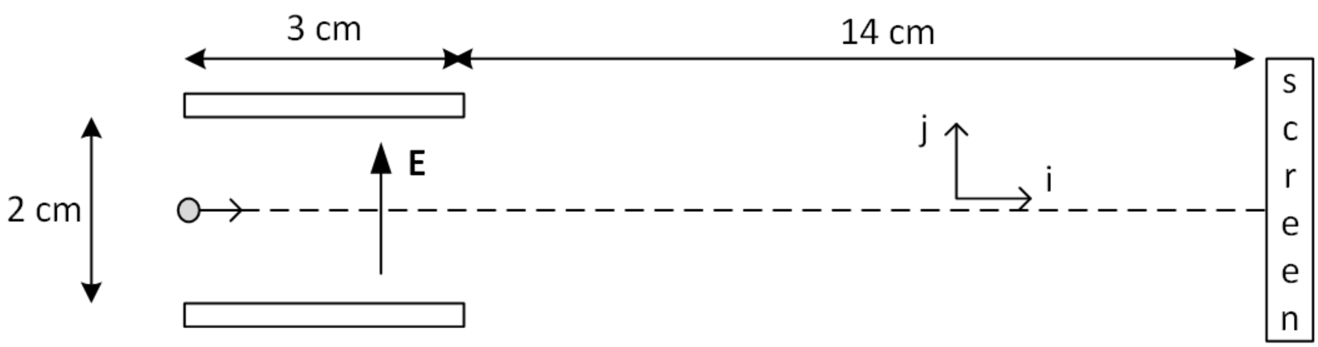
For positions $x > 0.50$ m, the total potential is always negative.

Thus, the only positions where the potential is 0 are at $x = -0.25$ m and $x = 0.125$ m.

c) In part (a) the point where $\vec{E}$ is zero must be on the line passing through the two charges. Explain why this is so. In part (b) do the point(s) have to be on the line passing through the charges? Explain.

The electric field must be on the line given that all the components must cancel. Should you go a little higher or a little lower, it would be impossible for the 2 vectors to be in opposite directions and cancel out. For potential they don't have to be. Given that it is a scalar, you just need the magnitudes to be opposite. That can happen in many places. In fact, there is a closed loop that can be drawn of the 0V equipotential surface.

3. An electron is projected along an axis midway between two opposite charged parallel plates with an initial velocity 2.00×10^7m/s $\hat{\imath}$. The field between the plates is 2.00×10^4 N/C $\hat{\jmath}$. Using the dimensions given in the figure, answer the questions which follow it.



a) How long is the electron in the field between the plates before emerging?

Since the electric field is in $\hat{\jmath}$, the force and acceleration of the electron are in $-\hat{\jmath}$. This means that the x-component of the electron's velocity is constant

$$t_{plates} = \frac{\Delta x}{v_x} = \frac{0.03}{2.00 \times 10^7} = 1.50 \times 10^{-9} s$$

b) What is the vertical displacement during this time?

The vertical acceleration of the electron is

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(-1.60 \times 10^{-19})(2.00 \times 10^4)}{9.11 \times 10^{-31}} = -3.51 \times 10^{15}\ m/s^2$$

The vertical displacement is

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(-3.51 \times 10^{15})(1.50 \times 10^{-9})^2 = -3.95 \times 10^{-3} m$$

c) With what velocity does the electron emerge from the field?

$$v_y = v_{0y} + at = 0 + (-3.51 \times 10^{15})(1.50 \times 10^{-9}) = -5.27 \times 10^6\ m/s$$

Since the x-component of the velocity has stayed constant,

$$\vec{v} = (2.00 \times 10^7 \hat{\imath} - 5.27 \times 10^6 \hat{\jmath}) m/s$$

d) At what vertical distance from its original axis will the electron strike the screen?

During this motion the electron feels no force and thus no acceleration. Repeating the process, the time to the screen is

$$t_{screen} = \frac{\Delta x}{v_x} = \frac{0.14}{2.00 \times 10^7} = 7.00 \times 10^{-9} s$$

The total vertical deflection of the electron is

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2 = -3.95 \times 10^{-3} + (-5.27 \times 10^6)(7.00 \times 10^{-9}) + 0$$
$$= -4.08 \times 10^{-2} m$$

The total deflection of the electron is 4.08 cm downward

4. A charged particle, q = 3.10 μC, is held in a fixed position. A second charge, Q, has the same charge as q and also a known mass of 2.00×10^{-5} kg. If this charge Q is initially held at rest at a distance 9.00×10^{-4} m from q, and then released from rest, what will be its speed at the instant it is 2.50×10^{-3}m from q?

We can solve this problem using conservation of energy,

$$U_i + K_i = U_f + K_f$$

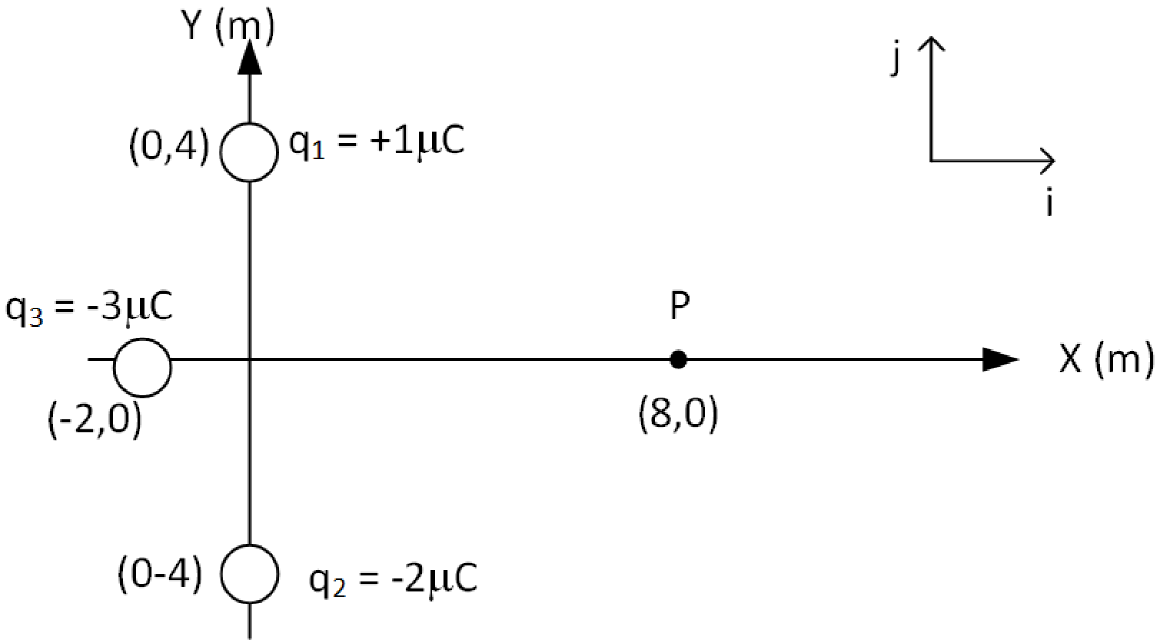
$$\frac{kQq}{r_1} + 0 = \frac{kQq}{r_f} + \frac{1}{2}mv^2$$

$$v^2 = \frac{2}{m}\left(\frac{kQq}{r_i} - \frac{kQq}{r_f}\right)$$

$$v = \sqrt{\frac{2Qq}{m}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)} = \sqrt{\frac{2(8.99 \times 10^9)(3.10 \times 10^{-6})^2}{(2.00 \times 10^{-5})}\left(\frac{1}{9 \times 10^{-4}} - \frac{1}{2.50 \times 10^{-3}}\right)}$$
$$= 2.48 \times 10^3 m/s$$

The charge Q's speed will be 2.48×10^3 m/s

5. The figure shows a system of three fixed point charges.



a) What is the total electric potential energy stored in the system of fixed point charges?

We simply need to take the sum of the potential energies between each pair of charges

$$U_{12} + U_{13} + U_{23} = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

$$= 8.99 \times 10^9 \left(\frac{(1 \times 10^{-6})(-2 \times 10^{-6})}{(2 \times 4)} + \frac{(1 \times 10^{-6})(-3 \times 10^{-6})}{\sqrt{2^2 + 4^2}} + \frac{(-2 \times 10^{-6})(-3 \times 10^{-6})}{\sqrt{2^2 + 4^2}} \right)$$

$$= 3.78 \times 10^{-3}\, J$$

b) What is the electric potential at the point P due to the system?

We add up the potential at point P from each charge, at distances $r_{1p} = r_{2p} = 8.9\, m$ and $r_{3p} = 10\, m$ found in problem 1.

$$V_p = V_1 + V_2 + V_3 = \frac{kq_1}{r_{1p}} + \frac{kq_2}{r_{2p}} + \frac{kq_3}{r_{3p}}$$

$$= 8.99 \times 10^9 \left(\frac{1 \times 10^{-6}}{8.9} + \frac{-2 \times 10^{-6}}{8.9} + \frac{-3 \times 10^{-6}}{10} \right)$$

$$= -3.71 \times 10^3\, V$$

c) What would be the electric potential energy of a charge $q_4 = -4.00 \times 10^{-10}$ C placed at P?
Since we've just found the potential at that point, we can multiply by the new charge

$$U = q_4 V_p = (-4 \times 10^{-10})(-3.70 \times 10^3) = 1.48 \times 10^{-6}\, J$$

d) What is the electric potential at the origin due to the three point charges?

Like in part b, we sum up the potential due to each charge

$$V_0 = \frac{kq_1}{r_{10}} + \frac{kq_2}{r_{20}} + \frac{kq_3}{r_{30}} = 8.99 \times 10^9 \left(\frac{1 \times 10^{-6}}{4} + \frac{-2 \times 10^{-6}}{4} + \frac{-3 \times 10^{-6}}{2} \right) = -1.57 \times 10^4\, V$$

e) What work is done by an external agent in moving q_4 from the point P to the origin with no change in speed?

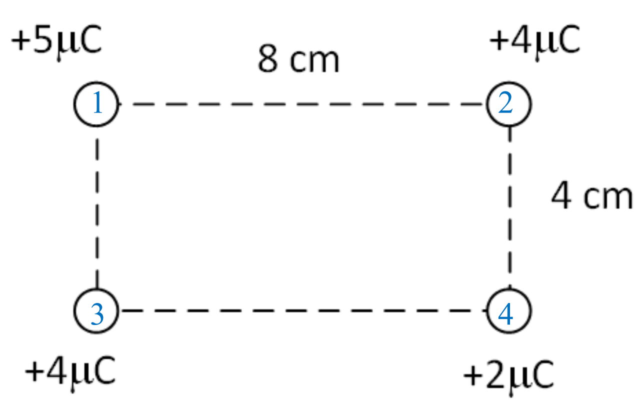
Using conservation of energy,

$$U_p + K_p + W = U_0 + K_0$$

Since there is no change in speed or kinetic energy,

$$\begin{aligned} W &= U_0 - U_p = q_4 (V_0 - V_p) \\ &= (-4 \times 10^{-10})(-1.57 \times 10^4 + 3.71 \times 10^3) \\ &= 4.80 \times 10^6\, J \end{aligned}$$

6. Four point charges are located at the corners of a rectangle as shown in the figure below. How much energy would be required to move the two 4.00 μC charges from their corners to infinity?



Since we are looking for the work spent between two different charge configurations, we can take the difference in the potential energies when all 4 charges are present compared to when only 2 charges are present. The difference in those energies will be the cost of moving the 2 charges away.

If we calculate the distance between the charges in opposite corners, we get

$$r_{14} = r_{23} = \sqrt{8^2 + 4^2} = 8.9\, cm$$

$$\begin{aligned} U_i &= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \\ &= \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_1q_4}{r_{14}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_2q_4}{r_{24}} + \frac{kq_3q_4}{r_{34}} \\ &= (8.99 \times 10^9)(10^{-6})^2 \left(\frac{5\cdot 4}{0.08} + \frac{5\cdot 4}{0.04} + \frac{5\cdot 2}{0.089} + \frac{4\cdot 4}{0.089} + \frac{4\cdot 2}{0.04} + \frac{4\cdot 2}{0.08}\right) \\ &= 12.0\,J \end{aligned}$$

$$U_f = U_{14} = \frac{kq_1q_4}{r_{14}} = \frac{(8.99 \times 10^9)(5 \times 10^{-6})(2 \times 10^{-6})}{0.089} = 1.0\,J$$

Thus, the work required to move the two 4.00 μC from the corners to infinity is

$$W = U_f - U_i = 1.0 - 12.0 = -11.0\,J$$

The answer is negative because the electric force does this work for us. No extra energy needs to be added to the system, the positive charges will repel each other if they can.

7. An electron moving parallel to the X-axis has a speed of 5.00×10^6 m/s at the origin. Its speed is reduced to 2.00×10^5 m/s at the point x = 4.00 cm.

a) What is the electric potential difference between the origin and the point x = 4.00 cm?

$$\Delta U = -\Delta K = q\Delta V$$

$$\begin{aligned} \Delta V = \frac{K_i - K_f}{q} &= \frac{m}{2q}\left(v_i^2 - v_f^2\right) = \frac{9.11 \times 10^{-31}}{2(-1.60 \times 10^{-19})}[(5.00 \times 10^6)^2 - (2.00 \times 10^5)^2] \\ &= -71.1\,V \end{aligned}$$

b) Which point is at the higher potential?

We just found a negative value for $\Delta V = V_f - V_i$, so the point x = 4.00 cm is at lower potential than the origin. We can validate this intuitively by seeing that the electron loses speed and thus kinetic energy. This means it gains potential energy, and since $U = qV$ the negative charge moves toward more negative potentials. We can imagine this as the electron moving away from a positive charge or towards a negative charge.