Problem Set #4 – Capacitors

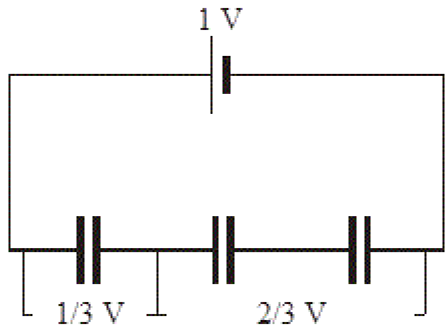
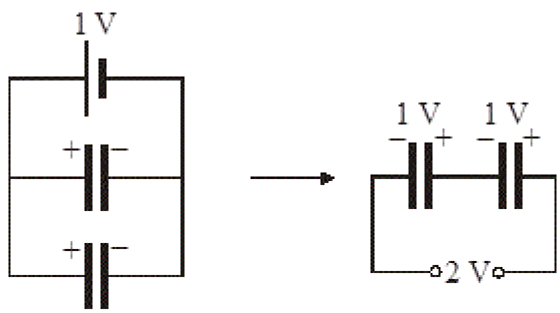
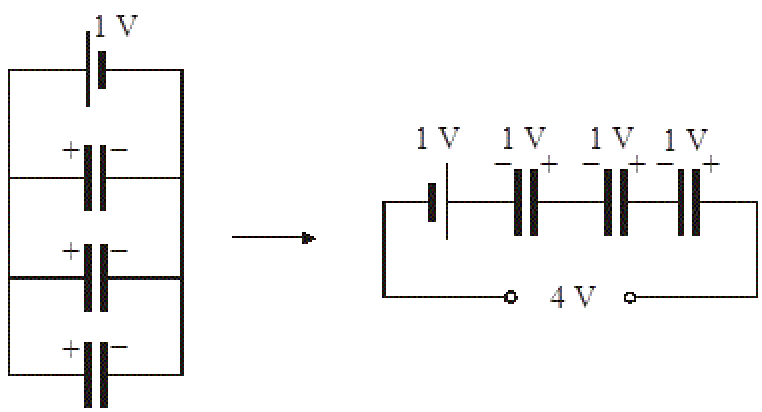
Solutions

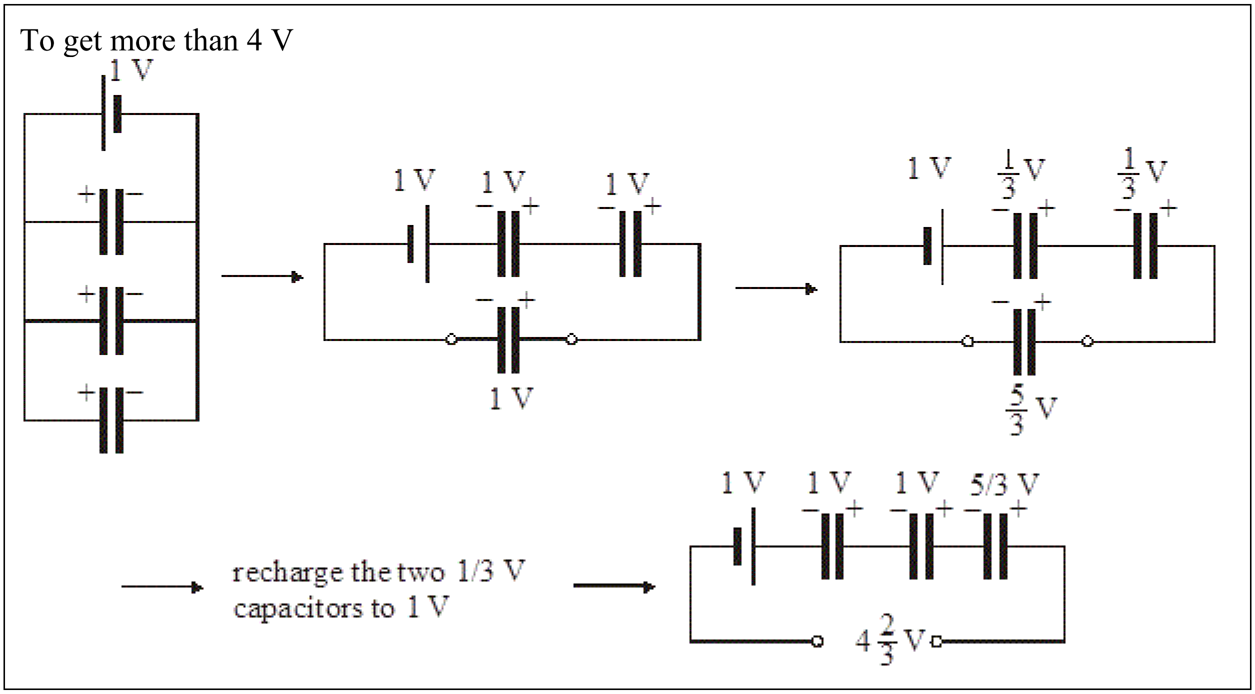
PART A: CONCEPTUAL QUESTIONS

(A) (d)

(B) (c)

(C)

To get 1/3 and 2/3 V:	
To get 2 V:	
To get 4 V:	



(D) (c)

PART B: NUMERICAL QUESTIONS

QUESTION 1

$Q = CV = 80.0 \times 10^{-9} \times 500.0 = \boxed{4.00 \times 10^{-5}\ \text{C}}$

QUESTION 2

(a) The equivalent capacitance of the 3.00-μF and 6.00-μF parallel combination is:

$$C_{eq,3,6} = 3 + 6 = 9.00\ \mu\text{F}$$

The equivalent capacitance of the 18.0-μF and 9.00-μF ($C_{eq,3,6}$) series combination is:

$$C_{eq,18,9} = \frac{18 \times 9}{18 + 9} = 6.00\ \mu\text{F}$$

The equivalent capacitance of the 4.00-μF and 12.0-μF series combination is:

$$C_{eq,4,12} = \frac{4 \times 12}{4 + 12} = 3.00\ \mu\text{F}$$

The equivalent capacitance of the $C_{eq,4,12}$ and $C_{eq,18,9}$ parallel combination, which is the equivalent capacitance of the circuit, is:

$$C_{eq} = 6 + 3 = \boxed{9.00\ \mu\text{F}}$$

(b) The 3.00-μF capacitor is part of the parallel combination $C_{eq,3,6}$. Since the voltage across that combination is 8.00 V, the charge on $C_{eq,3,6}$ is:

$$Q_{eq,3,6} = V_{eq,3,6}C_{eq,3,6} = 8.00\times 9.00\times 10^{-6} = 7.20\times 10^{-5}\text{ C}$$

Since that combination is in series with the 18.0-μF capacitor, the charge on the 18.0-μF capacitor is also $7.20\times 10^{-5}\text{ C}$. This means that the voltage across the 18.0-μF capacitor is:

$$V_{18} = \frac{7.2\times 10^{-5}}{18\times 10^{-6}} = 4.00\text{ V}$$

$$\Rightarrow V_{ab} = V_{18} + V_{eq,3,6} = 4.00 + 8.00 = 12.0\text{ V}$$

The charge on the 4-μF and 12-μF is the same and is equal to the charge on $C_{eq,4,12}$.

$$\Rightarrow Q_4 = Q_{eq,4,12} = V_{ab}C_{eq,4,12} = 12.0\times 3\times 10^{-6} = \boxed{36.0\ \mu\text{C}}$$

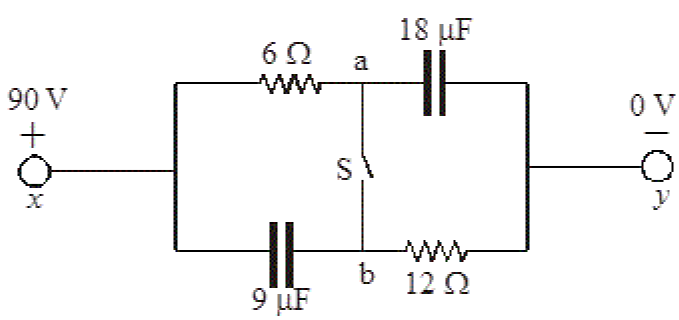
QUESTION 3

a) With the switch open, there is zero current through the circuit once the capacitors have become charged. Since the current is zero, the voltage drop across the resistors is also zero, i.e.

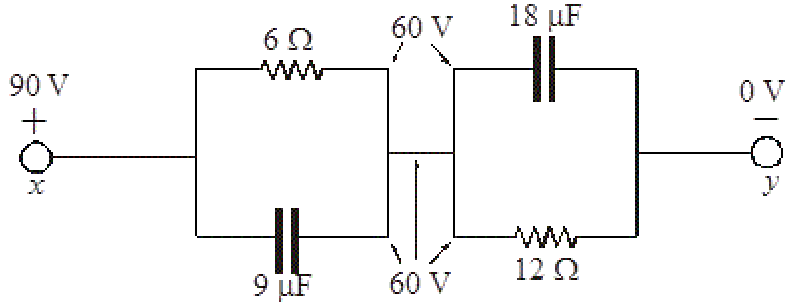
$V_a = 90\text{ V},\ V_b = 0\text{ V}$

For the 18 μF capacitor $\Rightarrow V_{ay} = V_a - V_y = 90 - 0 = \boxed{90.0\text{ V}}$

For the 9 μF capacitor $\Rightarrow V_{xb} = V_x - V_b = 90 - 0 = \boxed{90.0\text{ V}}$



b) With the switch closed, the circuit may be redrawn in the following way:



Current flows through the two resistors, which are now connected in series, and of course, no current flows through the capacitors. The voltage drop across the 6.00-Ω resistor is 30.0 V and the voltage drop across the 12.0-Ω resistor is 60.0 V. The potential difference across the 9.00-μF capacitor is $V_9 = 90.0 - 60.0 = \boxed{30.0\text{ V}}$

QUESTION 4

a) Since the capacitors are in series the charge is the same on both of them and is the same as on the equivalent capacitance that represents them. The equivalent capacitance of the series combination is: $C_{eq} = \dfrac{4\times 2}{4+2} = \dfrac{4}{3}\ \mu\text{F}$

The time constant of the charging circuit is: $\tau_C = R_C C_{eq} = 5.00\times 10^6 \times \dfrac{4}{3}\times 10^{-6} = \dfrac{20}{3}$ s

$$Q(t) = C_{eq}V\left(1-e^{-t/\tau_C}\right) = \frac{4}{3}\times 10^{-6}\times 20\times\left(1-e^{-\frac{10}{20/3}}\right) = \boxed{2.07\times 10^{-5}\text{ C}}$$

The current after 10 s of charge is: $I(t) = \dfrac{V}{R_C}e^{-t/\tau_C} = \dfrac{20.0}{5.00\times 10^6}e^{-\frac{10}{20/3}} = \boxed{8.93\times 10^{-7}\text{ A}}$

b) When the switch is flipped to B, the capacitors discharge through the 6.00-MΩ-3.00-MΩ parallel combination. The equivalent resistance of this combination is

$$R_D = \frac{3\times 6}{3+6} = 2.00\text{ M}\Omega$$

The time constant of the discharging circuit is: $\tau_D = R_D C_{eq} = 2.00\times 10^6 \times \dfrac{4}{3}\times 10^{-6} = \dfrac{8}{3}$ s

The charge on the capacitors after 10.0 s of discharge is:

$$Q(t) = C_{eq}Ve^{-t/\tau_D} = \frac{4}{3}\times 10^{-6}\times 20\times e^{-\frac{10}{8/3}} = \boxed{6.27\times 10^{-7}\text{ C}}$$

The voltage across the 4-μF capacitor at that time is:

$$V_4 = \frac{Q(10)}{C_4} = \frac{6.27\times 10^{-7}}{4.00\times 10^{-6}} = \boxed{0.157\text{ V}}$$

The current in the equivalent R after 10 s of discharge is:

$I(t) = \frac{V}{R_D} e^{-t/\tau_D} = \frac{20.0}{2.00\times10^6} e^{-\frac{10}{8/3}} = \boxed{2.35\times10^{-7}\ \text{A}}$ Therefore, the voltage across the equivalent R is $V = RI = 2.00\times10^6(2.35\times10^{-7}) = 4.70\times10^{-1}V$ so the current in the 3MΩ resistor will then be: $I(t) = \frac{V}{R} = \frac{4.70\times10^{-1}}{3.00\times10^6} = 1.57\times10^{-7}A$

QUESTION 5

a) Yes

b) Circuit 1 $I = \frac{V}{R} = \frac{40.0}{2.00} = 20.0A$

Circuit 2 $I = \frac{V}{R} = \frac{20.0}{8.00} = 2.50A$