SOLUTIONS: PROBLEM SET 2

ELECTRIC POTENTIAL AND ELECTRIC POTENIAL ENERGY

CONCEPTUAL QUESTIONS

A. It might be frightening to think you are at 5000V, but remember: potential is *energy per charge*. So if there's not a lot of charge, there's not really a lot of energy involved. Each coulomb of charge may have a potential energy of 5000J relative to the ground, but if you only have 10μC on your body, that's only 0.05J! The story is different if you have a source of charge that is significantly larger. Like a lightning bolt of Hydro-Quebec…

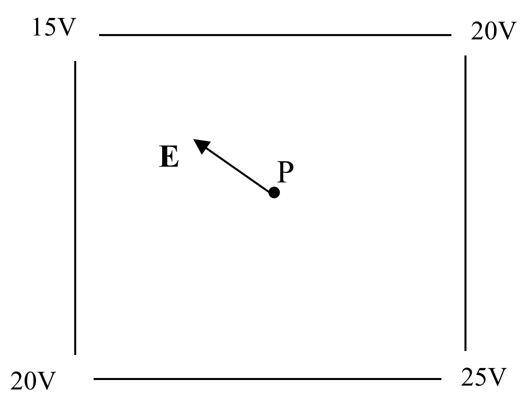
Consider an analogous situation with gravity: the gravitational potential at a point 500m above the ground is 5000J/kg (taking the ground as 0), but it makes a big difference if a grain of chalk dust is held over your head at that point or a car!

B. a)

C. b)

D. b)

E.



E = 1000 N/C = 1000 V/m

F. E (although D is pretty close)

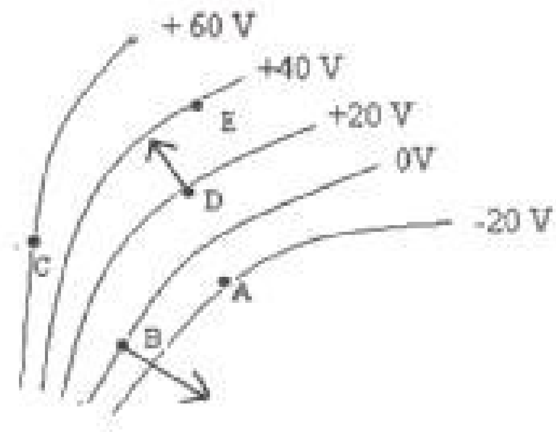
G. A

H. 80 V

I. C

J. 2x10^{-5}J **L.** (diagram)

K. 8x10^{-5}J **M**. 8E-5J **N**. 4x10^{-5}J



PROBLEMS

QUESTION 1

The potential energy between charges is given by: $U(r) = \frac{kq_1q_2}{r}$

We can find the distance between the charges

$$r = \frac{kq_1q_2}{U} = \frac{k(2.2\times10^{-6})(3.48\times10^{-6})}{0.4}$$
$$= 0.172\,m$$

b) The potential energy is related to kinetic energy: $\Delta K + \Delta U = 0$

$$K_f - K_i = U_i - U_f$$

$$\frac{1}{2}mv_f^2 - K_i = -(U_f - 0.4)$$

solve for the speed:

$$v = \sqrt{\frac{2(0.4)}{1.5\times10^{-5}}}$$

$$\boxed{v = 231\,m/s}$$

Note: What about if the two charges q_1 and q_2 were moving? **Try it!**

QUESTION 2

a) The electric potential energy needed to assemble this configuration of charges is the sum of all interactions, taken one pair at a time.

$$U(r) = \sum \frac{kq_i q_j}{r_{ij}} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$U = k\left(\frac{(2\times10^{-6})(-4\times10^{-6})}{0.04} + \frac{(2\times10^{-6})(3\times10^{-6})}{0.05} + \frac{(-4\times10^{-6})(3\times10^{-6})}{0.03}\right)$$

$$\boxed{U = -4.32\,J}$$

b) The electric potential of several charges is the algebraic sum of the individual contribution.

$$V_p = \sum \frac{kq_i}{r_i} = k\left[\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \frac{q_3}{r_{3p}}\right]$$

$$= k\left[\frac{2\times10^{-6}}{0.03} + \frac{-4\times10^{-6}}{0.05} + \frac{3\times10^{-6}}{0.04}\right]$$

$$\boxed{V_p = 5.55\times10^5 V}$$

c) The charge q at point P would have a potential energy

$$U = qV_p$$

$$\Delta K + \Delta U = 0$$

$$K_f - \cancel{K_i} = -(\cancel{U_f} - U_i)$$

$$\frac{1}{2}mv^2 = qV_p$$

$$\Rightarrow v = \sqrt{\frac{2qV_p}{m}}$$

$$= \sqrt{\frac{2(3\times10^{-10})(5.55\times10^5)}{4.2\times10^{-20}}}$$

$$\boxed{v = 8.90\times10^7 m/s}$$

QUESTION 3

The kinetic energy and potential energy are related to the electric potential.

$$\Delta K + \Delta U = 0 = qV_{BA}$$

$$K_f = K_i + \Delta K = K_i + (-\Delta U)$$

$$K_f = \frac{1}{2}(2.00E-4)(5.00^2) + [-(-5.00E-6)(800-200)]$$

$$K_f = (2.50E-3) + (3.00E-3)$$

$$K_f = 5.50E-3\ J$$

$$K_f = \frac{1}{2}mv_f^2 = 5.50\times 10^{-3} J$$

$$\Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

$$= \sqrt{\frac{2(5.50\times 10^{-3})}{2.00\times 10^{-4}}}$$

$$\boxed{v_f = 7.42 m/s}$$

QUESTION 4

The kinetic energy and potential energy are related to the electric potential

$$\Delta U = -\Delta K$$

$$qV_{3,0} = -(K_f - K_i)$$

$$\Rightarrow V_{3,0} = \frac{-(K_f - K_i)}{q} = -\left[\frac{(1.50\times 10^{-2})-(5.00\times 10^{-2})}{-6.50\times 10^{-7}}\right]$$

$$\boxed{V_{3,0} = -5.38\times 10^{4} V}$$

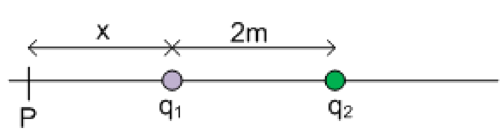
V at origin is 5.38×10^4 V higher than *V* at 3.00 cm.

Note: A positive charge going from 0 to 3.00 cm would lose potential energy and gain kinetic energy. The reverse happened for this negatively charged particle.

QUESTION 5

a) Three situations to consider:

- Situation 1: Consider point P, *x* meter to the left of q_1:



$$V_p = \sum \frac{kq_i}{r_i}$$

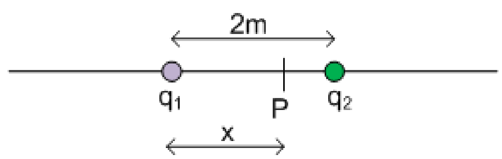
$$V_p = \frac{kq_1}{x} + \frac{kq_2}{(x+2.00)} = 0 \quad \rightarrow \quad \frac{q_1}{x} = -\frac{q_2}{(x+2.00)}$$

$$(q_1 + q_2)x = -2.00q_1$$

$$\Rightarrow x = -\frac{2.00(2.00\mu C)}{-1.00\mu C}$$

$$\boxed{x = 4.00m} \quad \text{to the left of } q_1$$

- Situation 2: Consider a different point P, in between q_1 and q_2



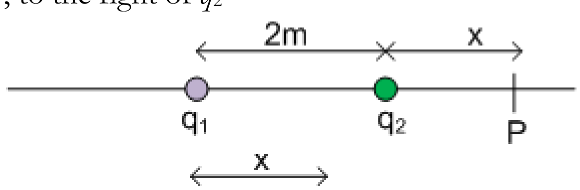
$$V_p = \frac{kq_1}{x} + \frac{kq_2}{(2.00-x)} = 0 \quad \rightarrow \quad \frac{q_1}{x} = -\frac{q_2}{(2.00-x)}$$

$$(-q_1 + q_2)x = -2.00q_1$$

$$\Rightarrow x = \frac{-2.00(2.00\mu C)}{-5.00\mu C}$$

$$\boxed{x = 0.80m} \quad \text{to the right of } q_1$$

- Situation 3: Consider point P, to the right of q_2



$$V_p = \frac{kq_1}{2.00+x} + \frac{kq_2}{x} = 0 \quad \rightarrow \quad \frac{q_1}{2.00+x} = -\frac{q_2}{x}$$

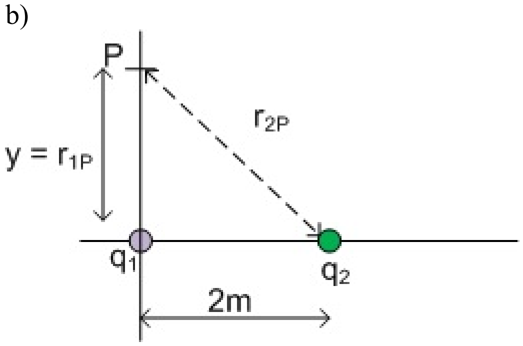
$$(q_1 + q_2)x = -2.00(q_2)$$

$$\Rightarrow x = \frac{-2.00(3.00\mu C)}{-1.00\mu C}$$

$$\boxed{x = -6.00m} \quad \text{to the right of } q_2 \text{ !?}$$

Answers: <u>4.00 m to the left of q_1 ad 0.80 m to the right of q_1</u>

b)



$$V_P = V_1 + V_2$$

The potential due to a point charge is $V = \frac{kQ}{r}$

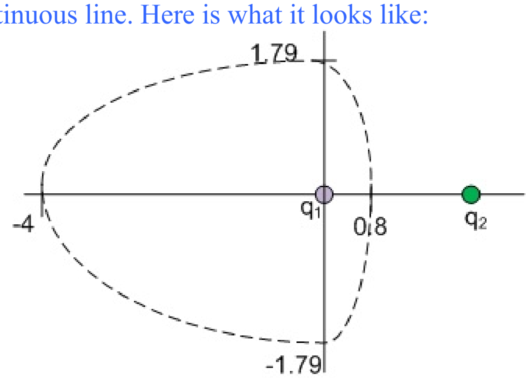
$$V_P = \frac{kq_1}{y} + \frac{kq_2}{\sqrt{2^2 + y^2}} = 0 \rightarrow \frac{q_1}{y} = -\frac{q_2}{\sqrt{4 + y^2}}$$

$$q_1^2(4 + y^2) = q_2^2 y^2$$

$$y^2(q_1^2 - q_2^2) = -4.00\, q_1^2$$

$$y = \sqrt{\frac{-4.00 \cdot (2.00\mu C)^2}{(2.00\mu C)^2 - (-3.00\mu C)^2}} = \pm 1.79\, m$$

<u>NOTE:</u> The equipotential $V = 0$ is a continuous line. Here is what it looks like:



c) No.

First of all, by inspection, it is clear that **E** cannot be 0 at any of the points except, maybe (-4, 0)m. and a quick calculation shows that it is not equal to 0 here as well. More generally, the value for V anywhere depend on the arbitrary choice of a reference zero and does not depend upon the **E** at the location in question. For example, a table top might be chosen as a level of 0 gravitational potential energy; that doesn't mean that the gravitational field is 0 at the level.

QUESTION 6

How close it will get? At the closest distance, the alpha particle will stop (before reversing it direction). If the speed is too high, than the particle will collide the nucleus. We cannot simply use kinematics because the acceleration is not uniform. However, we can use energy conservation principle!

The **energy conservation** states that:

$$(E_{tot})_{far} = (E_{tot})_{near}$$

$$Vq = U + \not{K} = \frac{kQq}{r}$$

$$r = \frac{kQ}{V} = \frac{k \cdot (79.0 \times 1.602 \times 10^{-19})}{3.00 \times 10^{6}} \Rightarrow \boxed{r = 3.79 \times 10^{-14}\, m}$$

The **force** according to Coulomb's law:

$$F = \frac{kqQ}{r^2} = \frac{k(2 \times 1.602 \times 10^{-19})(79 \times 1.602 \times 10^{-19})}{(3.79 \times 10^{-14})^2}$$

$$\boxed{F = 25.3N} \quad \textbf{WOW!}$$

The **acceleration**, according to Newton's second law:

$$a = \frac{F}{m} = \frac{25.3}{4 \times 1.67 \times 10^{-27}} \Rightarrow \boxed{a = 3.79 \times 10^{27}\, m/s^2}$$

QUESTION 7.

a) Recall your physics NYA and energy stored in a spring. The energy stored in a spring is given by $U = \frac{1}{2}kx^2$ (with k the spring constant). The energy conservation states that

$$(E_{tot})_A = (E_{tot})_B$$

$$0 = \frac{1}{2}kx^2 + (-V_{AB}q)$$

$$V = \frac{kx^2}{2q} = \frac{2.00(0.05)^2}{2(6.00\times10^{-5})} \quad \Rightarrow \quad \boxed{V = 41.7V}$$

b) The electric field is related to the electric potential if the electric field is uniform by:

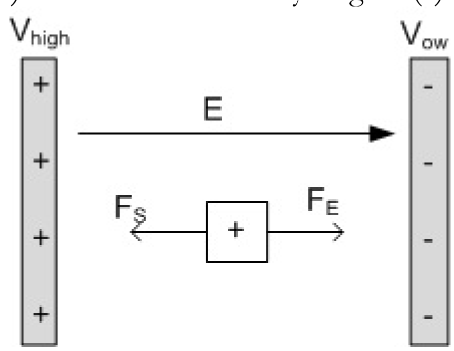
$$E = \frac{\Delta V}{\Delta x} = \frac{41.7}{0.05} \quad \Rightarrow \quad \boxed{E = 834V/m}$$

c) for a uniform electric field:

$$\Delta V = Ed = 834V/m \times 0.200\,m$$

$$\boxed{V = 167V}$$

d) Let's draw a free body diagram(!!)



$$\Sigma F = F_E - F_S = 0 \qquad \rightarrow F_E = F_S$$

$$qE = k\Delta x \qquad \rightarrow \Delta x = \frac{Eq}{k} = \frac{834V/m \cdot 6\times10^{-5}\,C}{2\,N/m} = \boxed{0.025m}$$

Answer: 2.50 cm. Note that this is the midpoint of the block's oscillation.