CONCEPTUAL QUESTIONS

A.

i.

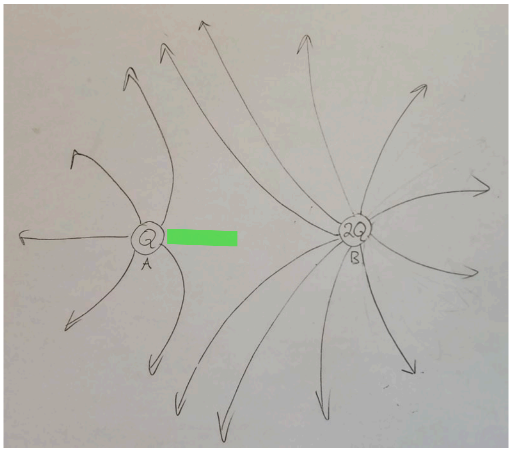


Figure 1: Electric field line diagram of charges A and B, where B is twice as strong as A. The green highlighted area shows the region in which there is a location where a third charge would experience zero net electric force.

Things to note about Figure 1:

- Direction given for every line: Direction is away from both positive charges
- Field lines meet each sphere at 90 degrees and never cross.
- The ratio of field lines is the same as the ratio of their charge 5:10 = 1:2.
- B is stronger so the lines leaving B take up more space than the lines leaving A.

ii. A third charge will feel no electric force at a position where there is no net electric field. This is somewhere along the green region highlighted above on the axes between the two charges. To show why, do the following:

- Draw the electric field vectors* from each of A and B at the 4 different possible regions: left of AB, between AB but closer to A, between AB but closer to B, and right of AB.

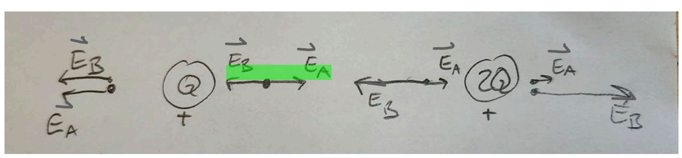
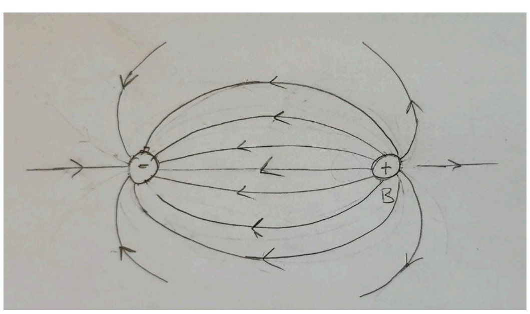


Figure 2: The field vectors of A and B at the 4 regions along the axis separating A & B..

- From this vector diagram, we can see that the only place where the two field vectors can possibly cancel out is in between both charges but closer to A, the weaker charge.
  *Instead of drawing 4 vector diagrams, you could draw 4 FBDs showing $\overrightarrow{F_A}$ and $\overrightarrow{F_B}$ on on a third charge (+ or -). The end result would be the same.

iii.



Things to note about Figure 3:

- Direction is given for every line and is away from + and towards -.
- Field lines meet each sphere at 90 degrees and never cross.
- The ratio of field lines is equal, like their charges: 10:10 = 1:1.
- The charges have equal strength and so the diagram is symmetrical.

Figure 3: Electric field line diagram of charges A and B, where B is twice as strong as A. The green highlighted area shows the region in which there is a location where a third charge would experience zero net electric force.

There is NO LOCATION where a third charge would experience zero net electric force, because there is no location where the net field is zero. To show why, draw the two field vectors for each possible region: left of AB, between AB, and right of AB.

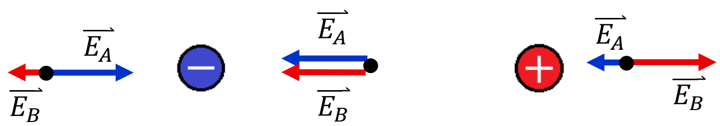


Figure 4: The field vectors from A and B at the three possible regions along their axis.

Figure 4 shows that outside of both charges the fields point in opposite direction BUT one will always be bigger so they can never cancel out. Inside of both charges, the field always points to the left so they can never cancel. There is therefore no regions where the net field is zero.

B. c) down.

To understand why, draw the field vectors at point P, for a few points from both the positive and the negative sides. Make sure you choose symmetrical positions to see how they combine. You will find the horizontal (left and right) components cancel each other out, but the vertical components combine in the downward direction.

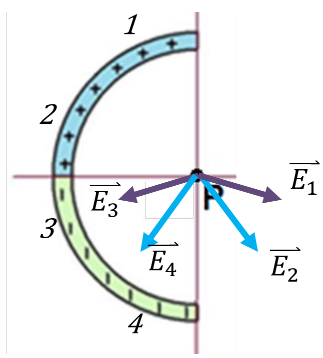


Figure 6: Field vectors at point P, generated by the charge at locations 1, 2, 3 and 4 .

C. d) it is non-uniform.

A uniform field causes constant acceleration, resulting in a parabolic trajectory. This trajectory is not parabolic, therefore the acceleration is not constant and the field is not uniform. According to the sketch, the negative charge's acceleration first points up and later down. That means the electric field points down in the left area, changing to up on the right side.

D. a) the trajectory is unchanged.

The acceleration from an electric field is proportional to the charge to mass ratio: $\vec{a} = \vec{E}\frac{q}{m}$. In this situation, both q and m are doubled, so the ratio doesn't change, and neither does the acceleration. The trajectory will not change. Importantly, the charge remains negative. If the charge had changed signs, what would happen to its acceleration and trajectory?

E. **e)** $\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$

This is the Third Law of Motion and can not be broken. It doesn't matter that they have different charges. Remember, that the force between them is proportional to the product of BOTH of their charges. So if one has a bigger charge, they will both experience a bigger force, but no matter what, the forces between them are equal and opposite.

F. **No, it is not possible.** Electric field is the force per charge: $\vec{E} = \frac{\vec{F}}{q}$. If the field is zero, then by definition, there is zero electric force on a charge at that position.

G. **e) In quadrant IV:** $270° < \theta < 360°$

To approximate the direction of the next force, draw a FBD on the particle on which the force is acting.

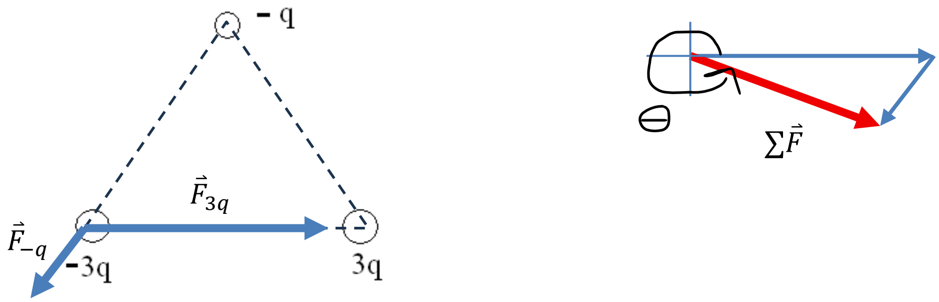


Figure 7: FBD of both electric forces on the -3q charge. From this, you can see that the net force will point down and to the right, which is in the fourth quadrant.

H. To create a uniform electric field, we need two plates of opposite charge, where the separation distance between them is much smaller than the length of the plates. Requirements are:

- d<< L.
- Field lines are parallel and evenly spaced except at ends, pointing from + to -.
- Field lines touch charged plates at 90°
- Near edges, the lines curve out and become less dense but still meet plates at 90°.

Figure 8 is an example. I've chosen 0.5 cm << 30 cm, but anything reasonable is fine.

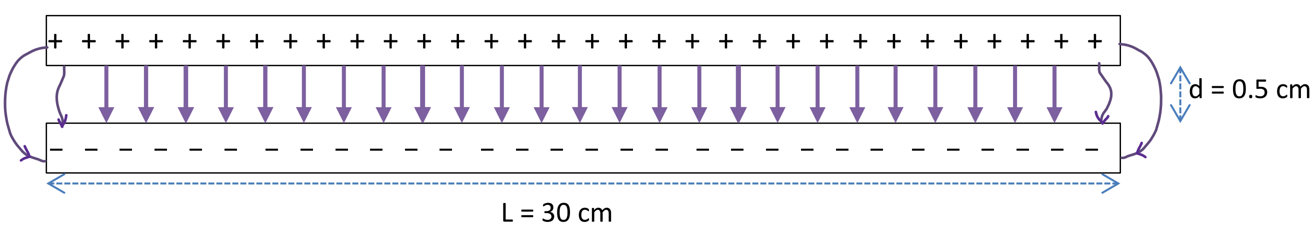


Figure 8: configuration of charges that will produce a relatively uniform electric field.

PROBLEMS

QUESTION 1

a) Draw a FBD (Figure 9). The electric force must point up to counteract the downward gravitational force. This means the electric field must point down (remember, the drop is negatively charged).

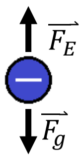


Figure 9: FBD of negatively charged oil drop in static equilibrium.

b) $q_{drop} = -3e = -4.8\mathbf{0} \times 10^{-19}\ C.$

Since the charge is at equilibrium, the electric force and gravitational force are equal and opposite.

- $\vec{F}_E = -\vec{F}_G$
- $F_E = F_G$
- $|q|E = \text{mg}$
- $4.80 \times 10^{-19}(8.17 \times 10^5) = 9.81\text{m}$
- $\mathbf{m = 4.00 \times 10^{-14}}\ \boldsymbol{kg}$

c) If the mass increases, the gravitational force increases while the electric force remains unchanged. The new FBD would look like Figure 10, resulting in a downwards acceleration. **The drop will fall down.**

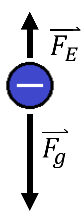


Figure 10: FBD of oil drop when mass is increased.

d) If the mass stays the same* instead, and the negative charge becomes stronger, the electric force increases while the gravitational force remains unchanged. *the mass of some excess electrons (x10^{-31} kg) is insignificant compared to mass of the oil drop (x10^{-13} kg). This results in the **drop having an upward acceleration**.

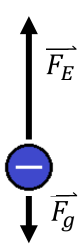


Figure 11: FBD of oil drop when charge is increased.

QUESTION 2

a) We have two unknowns (q_A and q_B), so we will need two unique physical relationships between them.

The force each charge experiences from the other is always equal in magnitude.

- $|\vec{F}_{\mathrm{AB}}| = |\vec{F}_{\mathrm{BA}}| = 1.00\ \mathrm{N}$

The magnitude of the force between two point charges can be calculated using Coulomb's Law:

- $1.00 = (8.99 \times 10^9)\frac{|q_A q_B|}{2.00^2}$ → *Ignore absolute value since both charges are +'ve. Now let's isolate one of the charges.*
- $q_A = \frac{4.45\times10^{-10}}{q_B}$ **(Equation 1)**

We also know the sum of the two charges:

- $q_A + q_B = 5.00 \times 10^{-5}\ C$ **(Equation 2)**

Substituting Equation 1 into Equation 2 gives:

- $\frac{4.45\times10^{-10}}{q_B} + q_B = 5.00 \times 10^{-5}$
- $4.45 \times 10^{-10} + {q_B}^2 = (5.00 \times 10^{-5})q_B$ → *Solve this quadratic equation*
- $q_B = 3.84 \times 10^{-5}\ C\ or\ 1.16 \times 10^{-5}\ C$ → *q_B must be the weaker one and q_A is the stronger*

$q_A = \mathbf{3.84 \times 10^{-5}\ C}$, $q_B = \mathbf{1.16 \times 10^{-5}\ C}$

b) To neutralize A, we must add as many electrons as it takes to equal $-q_A$. The charge of an electron is $-e$.

- $\#\ \boldsymbol{electrons} = \frac{-3.84\times10^{-5}\ C}{-e} = \mathbf{2.40 \times 10^{14}}\ electrons$

QUESTION 3

Draw the electric field vector at R, S and T. Remember:

- A negative charge creates an electric field pointing towards itself.
- The field strengths at R & S are the same because R & S are equidistant from the source charge.
- Position T has a weaker electric field because it is further from the source charge.

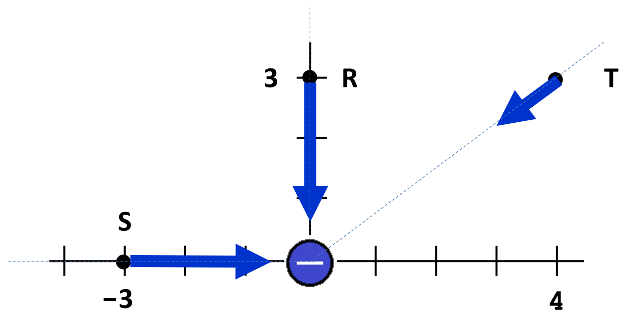


Figure 12: Field vectors at each position, with source charge at the origin.

For each position, calculate the strength of the electric field at R, using the equation for the field due to a point charge, using your diagram (Figure 12) to ascertain the direction.

- $E_R = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{4 \times 10^{-6}}{0.03^2} = 4.00 \times 10^7 N/C$ → $E_R = -4.00 \times 10^7\ \hat{\jmath}\ N/C$
- $E_S = E_R = 4.00 \times 10^7 N/C$ → $E_S = +4.00 \times 10^7\ \hat{\imath}\ N/C$
- $E_T = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{4 \times 10^{-6}}{(.03^2 + .04^2)} = 1.44 \times 10^7 N/C$ → *Need a diagram to find the polar angle*

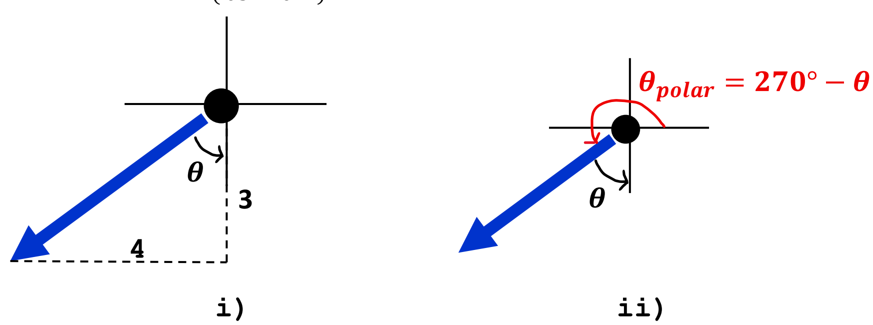


*Figure 13: **Sketch i)** shows the triangle that can be used to calculate the angle between T's electric field vector and the -y axis, θ. **Sketch ii)** shows the full polar angle and its relationship with the smaller angle, θ.*

Use your diagram (Figure 13) to calculate the polar angle of T's electric field vector:

- $\theta_{polar} = 270° - tan^{-1}\left(\frac{4}{3}\right)$
- $\theta_{polar} = 270° - 53.1° = 216.9°$ → *Use polar angle to put in component form.*
- $E_T = 1.44 \times 10^7 (cos216.9°\hat{\imath} + sin216.9°\hat{\jmath})$ → $E_T = (-1.15E7\hat{\imath} - 8.65E6\hat{\jmath})\ N/C$

QUESTION 4

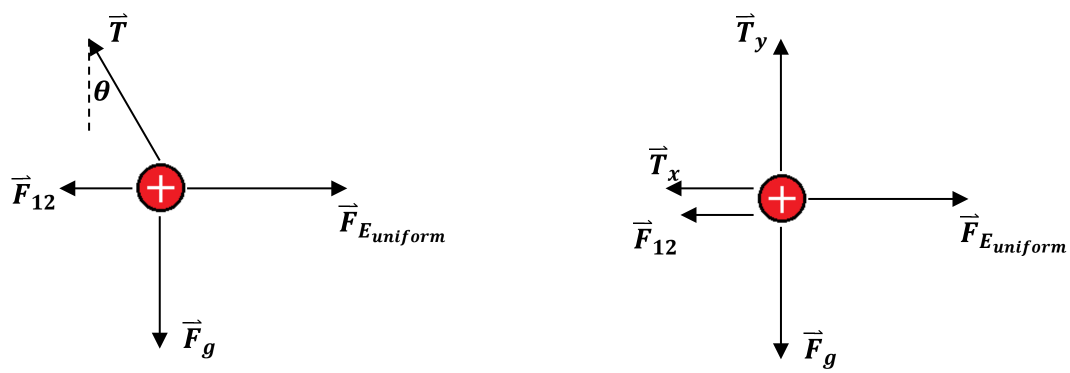
The force and the electric field are related by the definition of field as the force per charge: $\vec{E} = \frac{\vec{F}}{q} \rightarrow \vec{F} = q\vec{E}$

- $\vec{F}_R = (-e)\vec{E}_R = -1.60 \times 10^{-19} \cdot (-4.00 \times 10^7 \hat{\jmath}) = +6.40 \times 10^{-12}\, \boldsymbol{\hat{\jmath}}\, N$
- $\vec{F}_S = (-e)\vec{E}_S = -1.60 \times 10^{-19} \cdot (4.00 \times 10^7 \hat{\imath}) = -6.40 \times 10^{-12}\, \boldsymbol{\hat{\imath}}\, N$
- $\vec{F}_T = (-e)\vec{E}_T = -1.60 \times 10^{-19} \cdot (-1.15 \times 10^7 \hat{\imath} - 8.65 \times 10^6 \hat{\jmath}) = (1.84\hat{\imath} + 1.38\, \hat{\jmath}) \times 10^7\, N$

Before you go: Are the directions of these vectors what you expected? If the test charge placed in the E-field is positive, what would happen to the direction and magnitude of the force acting on it?

QUESTION 5

As always, draw a FBD. Either charge will do so I'll draw the +'ve charge ($q_2 = 5.00 \times 10^{-8}\, C$).



i) All forces acting on q_2. **ii) With tension in components**

Figure 14: FBD of positive charge.

The charge is in equilibrium so we can balance forces along both axes. Start with the simplest: the y-direction.

- $\sum F_y = 0$
- $T_y - F_g = 0$
- $T\cos\theta - mg = 0$ → *Use FBD (sketch i) to understand why the y-component of tension uses the cosine of θ°.*
- $T = \frac{mg}{\cos\theta}$ → ***Equation 1*** *(good practice to leave it in symbol form)*

Now that we have an expression for tension, let's balance forces along the x-axis.

- $\sum F_x = 0$
- $F_{E_{uniform}} - T_x - F_{12} = 0$
- $|q_2|E - \frac{mg}{\cos\theta}(\sin\theta) - k\frac{|q_1 q_2|}{r^2} = 0$ → **Equation 2** (remember $T_x = T\sin\theta$ and Equation 1 is T)

Equation 2 can be solved for the uniform electric field strength E, if we know the distance between the two spheres, r. To find this, we should draw a diagram.

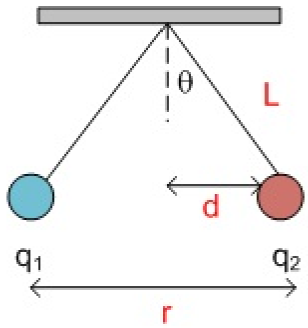
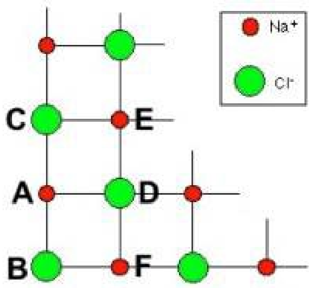


Figure 15: Geometry of hanging charges, showing that r = 2d, where d can be found using triangle trigonometry.

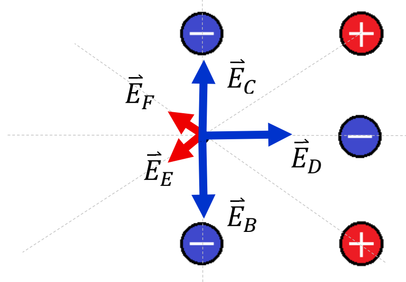
- r = 2d = $2Lsin\theta$ → *This is now ready to insert into Equation 2.*
- $qE - mgtan\theta - k\frac{|q|^2}{(2Lsin\theta)^2} = 0$ → *I've simplified the tension term using* $tan\theta = \frac{sin\theta}{cos\theta}$
- $5 \times 10^{-8}E - 0.002 \cdot 9.81 \cdot tan10° - 8.99 \times 10^9 \frac{|5\times10^{-8}|^2}{(2\cdot0.10\cdot sin10°)^2} = 0$
- $5 \times 10^{-8}E - 3.46 \times 10^{-3} - 2.51 \times 10^{-2} = 0$
- $E = 4.92 \times 10^5\ N/C$ → $\boldsymbol{\vec{E} = 4.92 \times 10^5\ \hat{\imath}\ N/C}$

QUESTION 6

a) To find the net force on ion A from its immediate neighbours, I'll first find the net electric field at A's position, then use that to find the net force. If you prefer, you can use Coulomb's Law to find the net force directly. Both approaches make sense. But first… a diagram!



i) one edge of one plane of the crystal



ii) Electric field vectors at ion A's location.

Figure 16: Simplification of crystal to explore the effects of the other electric fields on the edge ion A.

Use Figure 13 to find the direction and strength of the electric field of each of the negative ions B, C ,& D.

- $|\vec{E}_B| = |\vec{E}_C| = |\vec{E}_D| = \frac{ke}{d^2}$
- $\vec{E}_B = -\frac{ke}{d^2}\hat{j},\ \ \vec{E}_C = +\frac{ke}{d^2}\hat{j},\ \ \vec{E}_C = +\frac{ke}{d^2}\hat{\boldsymbol{\imath}}$

Use Figure 13 to find the direction and strength of the electric field of each of the positive ions E & F.

- $|\vec{E}_E| = |\vec{E}_F| = \frac{ke}{2d^2}$
- $\vec{E}_E = \frac{ke}{2d^2}(cos225°\hat{\boldsymbol{\imath}} + \boldsymbol{sin}225°\hat{\boldsymbol{\jmath}}),\ \ \vec{E}_F = \frac{ke}{2d^2}(cos135°\hat{\boldsymbol{\imath}} + \boldsymbol{sin}135°\hat{\boldsymbol{\jmath}})$

Now sum all field vectors to find the net electric field that will exert a force on ion A. Notice that the vertical components of B and C, as well as E and F cancel out as shown in Figure 16.

- $\sum\vec{E} = -\cancel{\frac{ke}{d^2}\hat{j}} + \cancel{\frac{ke}{d^2}\hat{j}} + \frac{ke}{d^2}\hat{\boldsymbol{\imath}} + \frac{ke}{2d^2}(cos225°\hat{\boldsymbol{\imath}} + \cancel{\boldsymbol{sin}225°\hat{\boldsymbol{\jmath}}}) + \frac{ke}{2d^2}(cos135°\hat{\boldsymbol{\imath}} + \cancel{\boldsymbol{sin}135°\hat{\boldsymbol{\jmath}}})$
- $\sum\vec{E} = \left(\frac{ke}{d^2} + \frac{ke}{2d^2}cos225° + \frac{ke}{2d^2}cos135°\right)\hat{\boldsymbol{\imath}}$ → *We know* $cos45° = |cos135°| = |cos225°| = \frac{1}{\sqrt{2}}$
- $\sum\vec{E} = \left(\frac{ke}{d^2} - \frac{ke}{2d^2}\frac{1}{\sqrt{2}} - \frac{ke}{2d^2}\frac{1}{\sqrt{2}}\right)\hat{\boldsymbol{\imath}} = \left(\frac{ke}{d^2} - \frac{ke}{\sqrt{2}d^2}\right)\hat{\boldsymbol{\imath}}$
- $\sum\vec{E} = \frac{ke}{d^2}\left(1 - \frac{1}{\sqrt{2}}\right)\hat{\boldsymbol{\imath}}$ ***Equation 1***

The net field given in Equation 1, points to the right, into the crystal. Ion A is a positive ion, and so will experience a net force also to the right, into the crystal.

- $\sum\vec{F} = \frac{ke^2}{d^2}\left(1 - \frac{1}{\sqrt{2}}\right)\hat{\boldsymbol{\imath}}$

b) Ion A experiences a force that pulls it into the crystal, thus helping to hold the crystal together. Our analysis has showed that the attractive forces are stronger than the repulsive forces, resulting in the ions being pulled towards each other, not forced apart. This shows that the electric forces help to hold the crystal together.

c) Yes, this approach is reasonable. We have only considered the nearest ions because the force is strongest along the shortest distances between the atoms in the crystal. At other distances, the force is smaller and won't have enough of an effect to change the direction of the net force on any given ion: $\vec{F} \propto \frac{1}{r^2}$.