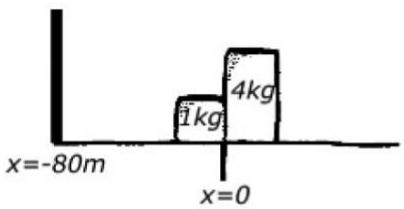
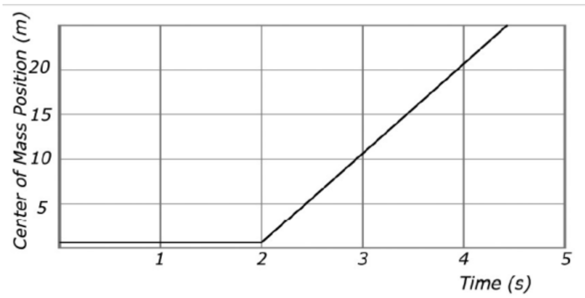
PROBLEM SET #6
Impulse, linear and angular momentum

Conceptual Questions



Question1. Two blocks of mass 1-kg and 4-kg are placed in contact at the centre point of an 80-m long track that exerts negligible friction on the blocks. The blocks are exploded apart. There is a wall located at $x = -80\ m$. Upon striking the wall, the 1-kg block sticks to the wall. The graph of the centre of mass position of the two-block system is shown as a function of time.

The graph shows the position of the centre of mass of the two block-system as function of time. The graph is zero for $0 < t < 2\ s$, and has a constant positive slope for $t > 2$ s. Explain why the graph has these features.



Answer:

Because there is no net external force on the two-block system from time 0 < t < 2 seconds, the center of mass of the system
remains at rest. At t = 2 seconds,
there is a net external force (an impulse) applied to the system by the wall. The center of mass has to remain in the same location relative to the two masses, namely the ratio of distances from the center of mass has to be 4 to 1. So, after the left mass stops and the right mass keeps going, the center of mass has to move to the right to keep the ratio at 4 to 1. Also, since there were no net external forces exerted on the system from time 0 < t < 2 seconds, the center of mass of the system remained at rest until the system received an impulse from a net external force (the wall), at which point the center of mass travels to the right at a constant speed because it was initially given an instantaneous force to the right during the explosion at time t = 0 seconds.

Question 2. Two students are arguing about this, talking about a collision between object 1 and object 2:

Kalil: Conservation of momentum says that the total momentum of the system before the collision equals the total momentum of the system after the collision:

$$m_1\vec{v}_{1before} + m_2\vec{v}_{2before} = m_1\vec{v}_{1after} + m_2\vec{v}_{2after}$$

Jennifer: No, I think conservation of momentum says that, in the collision, the momentum gained by one object equals the momentum lost by the other:

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

The minus sign says that the *gain* in object 1's momentum equals the *loss* in object 2's momentum, or vice versa; one compensates for the other.

Which student or students do you agree with?

a) I agree with Kalil.
b) I agree with Jennifer.
c) I agree with both: They're saying the same thing in different ways.
d) I agree with neither.

Answer:

Kalil:

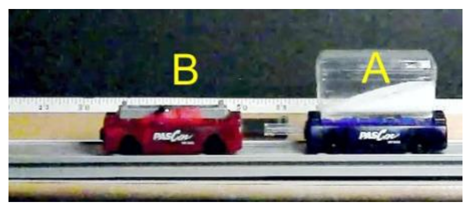
$$-m_2\vec{v}_{2after} + m_2\vec{v}_{2before} = m_1\vec{v}_{1after} - m_1\vec{v}_{1before}$$

$$-(m_2\vec{v}_{2after} - m_2\vec{v}_{2before}) = m_1\vec{v}_{1after} - m_1\vec{v}_{1before}$$
$$-m_2\Delta\vec{v}_2 = m_1\Delta\vec{v}_1$$

Jennifer:

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

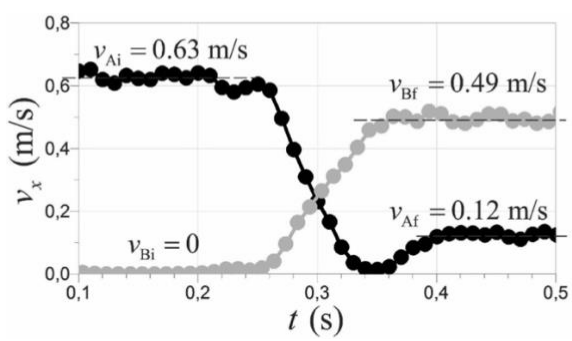
Question 3. You have two low-friction carts on a track. Cart A has a spring bumper that compresses and expands elastically. You adjust the total masses of the carts so they are both equal to 565 g.



You push cart A so that it starts moving towards cart B (which is initially at rest). The carts collide.

The figure below shows the velocity-versus-time graph for both carts during the collision. Average velocities of the carts before and after the collision are also shown on the graph.

In the following activities take both carts as the system.



a) Take the situation at t = 0.20 s as the initial state, and the situation at t = 0.45 as the final state. Can you say that the total momentum of the system is constant in this process? If not, explain what might have caused the total momentum change. Can you say that the total mechanical energy of the system is constant in this process? If not, estimate the change of the total mechanical energy and explain into what other forms of energy it was converted.

Answer:

$m_{1f}\,v_{1f} + m_{2f}\,v_{2f} < m_{1i}\,v_{1i} + m_{2i}\,v_{2i}$

As the total momentum of the system was not constant it means the net force acting on the system is not zero (not a closed system)

$\frac{1}{2}\,m_{1f}\,v_{1f}^2 + \frac{1}{2}\,m_{2f}\,v_{2f}^2 < \frac{1}{2}\,m_{1i}\,v_{1i}^2 + \frac{1}{2}\,m_{2i}\,v_{2i}^2$

Energy is not conserved. Some energy is transformed into other forms, such as heat or sound and deformation.

b) At t = 0.35 s, cart A almost stopped moving but after that time it sped up and continued moving with constant speed. Explain the mechanism that made the cart speed up.

Answer:

The mechanism that made cart A speed up again is the release of elastic potential energy stored in the compressed spring, which converts back into kinetic energy

c) Your friend notices that the magnitudes of the slopes of the velocity-versus-time curves for the carts during the collision are different. He argues: "Different slopes mean different accelerations during the collision. Because the masses of the carts are equal, this means that the magnitudes of the forces exerted by one cart on another are not equal during the collision. I think that, based on this experiment, we can reject Newton's third law."

Do you agree or disagree with your friend? If you disagree, find the mistakes in your friend's reasoning and show that the outcome and the data obtained in this experiment are consistent with Newton's laws.

Answer:

Disagree! The magnitudes of the net forces are different not the forces exerted by one cart on another.

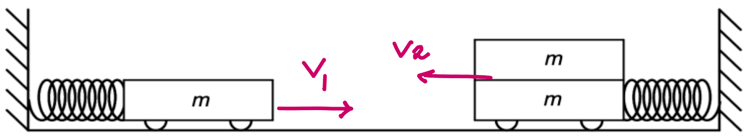
Question 4. A wood block rests at rest on a table. A bullet shot into the block stops inside, and the bullet plus block start sliding on the frictionless surface. The horizontal momentum of the bullet plus block remains constant

a) Before the collision.
b) During the collision
c) After the collision
d) All of the above
e) Only A and C above

Question 5. Two ball of different mass, m_1 and m_2, are pitched at the same velocity $\vec{v}_i$ to a baseball player, who bats each one, applying the same impulse to each ball. Which of the following quantities **must** be the same?

a) $\vec{p}_{1f}$ and $\vec{p}_{2f}$
b) $\vec{v}_{1f}$ and $\vec{v}_{2f}$
c) $\vec{F}_1$ and $\vec{F}_2$
d) $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$
e) $\vec{a}_1$ and $\vec{a}_2$

Question 6. Two carts, one mass m, the other mass $2m$, are compressed the same distance x against identical springs and released from rest. They separate from the springs and collide and stick together. In which direction will the two carts move after the collision?



a) Left
b) Right
c) The two carts will stop in the center.
d) The direction depends on the relationship between mg and kx.

After leaving the springs:

$$U_1 = U_2$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$$

$$m_2 = 2\, m_1$$

$$v_2 = \frac{\sqrt{2}}{2}\, v_1 = 0.7\, v_1$$

Perfectly inelastic collision:

$$m_1\vec{v}_{1before} + m_2\vec{v}_{2before} = (m_1 + m_2)\vec{v}_{after}$$

$$m\vec{v}_{1before} - 2m(0.7)\vec{v}_{1before} = (3m)\vec{v}_{after}$$

$$v_{after} = -\frac{0.4}{3}\,\vec{v}_{1before}$$

Direction: Left

Question 7. A figure skater rotating on one spot with both arms and one leg extended has moment of inertia I_i. She then pulls in her arms and the extended leg, reducing her moment of inertia to $0.75\ I_i$.What is the ratio of her final to initial kinetic energy (K_f/K_i)?

$I_f = 0.75\ I_i$
Conservation of angular momentum:
$I_i\ \omega_i = I_f\ \omega_f$
$\omega_f = (I_i\ /\ I_f)\ \omega_i = (1/0.75)\ \omega_i$
$K_f / K_i = (0.5\ I_f\ \omega_f^{\ 2}) / (0.5\ I_i\ \omega_i^{\ 2}) = (0.75\ I_i\ * ((1/0.75)\ \omega_i)^{\ 2}$
$K_f / K_i = 1.33$

Question 8. A mouse hangs on the rim of a spinning turntable. The rotational inertia of the mouse about the centre of the turntable is about the same as the rotational inertia of the turntable by itself. Mouse and turntable are freely spinning with constant velocity ω. The mouse starts to crawl toward the centre of the turntable. At a particular position the angular velocity of the mouse and the turntable has increased to 4ω

At this instant, what is the total angular momentum of the system in terms of its original value $|\vec{L}_i|$?

Answer:

No external torques act on the system of mouse and turntable, so as long as the mouse stays on the turntable, the total angular momentum cannot change.

Question 9. A woman sits on a stool that can turn friction-free about its vertical axis. She is handed a spinning bicycle wheel that has angular momentum $\vec{L}_o$ and she turns it over (that is, through 180°). She thereby acquires an angular momentum of magnitude…

a) 0 b) $\frac{1}{2}\vec{L}_o$ c) $\vec{L}_o$ d) $2\vec{L}_o$ e) $4\vec{L}_o$

$L_{before} = L_{after}$

$L_{before\ woman} + L_{before\ wheel} = L_{after\ woman} + L_{after\ wheel}$

$0 + L_o = L_{after\ woman} + (-L_o)$

$2L_o = L_{after\ woman}$

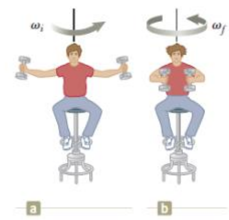
Question10. True or false? If the net torque on a rotating system is zero, the angular velocity of the system cannot change. If your answer is false, give an example of such a situation.

Answer:

The net torque is zero, all we can say for sure is that the angular momentum (the product of *I* and ω) is constant. If I changes, so must ω!

Consider a spinning ice skater who changes her body position while maintaining the same spin axis. Initially, the skater spins with arms extended and then pulls her arms in. The **net external torque** is still zero, so the angular momentum is conserved. However, the skater's angular velocity changes in magnitude due to a change in her moment of inertia. In this case, angular speed changes while torque remains zero.

Question 11. A student, with arms at her sides, is spinning on a frictionless turntable. When the student extends her arms,



a) her angular velocity increases.
b) her angular velocity remains the same.
c) her rotational inertia decreases.
d) her rotational kinetic energy increases.
e) her angular momentum remains the same.

Question 12. Two identical cylindrical discs have a common axis. Initially one of the discs is spinning. When the two discs are brought into contact, they stick together. Which of the following is true?

a) The total kinetic energy and the total angular momentum are unchanged from their initial values.
b) Both the total kinetic energy and the total angular momentum are reduced to half of their original values.
c) The total angular momentum is unchanged, but the total kinetic energy is reduced to half its original value.
d) The total angular momentum is reduced to half its original value, but the total kinetic energy is unchanged.
e) The total angular momentum is unchanged, and the total kinetic energy is reduced to one-quarter of its original value.

$I_{disk} = 0.5\ mr^2$
$m_f = 2m_i$
$I_f = 2\ I_i$
Conservation of angular momentum:
$I_i\ \omega_i = I_f\ \omega_f$
$\omega_f = (I_i\ /\ I_f)\ \omega_i = (0.5)\ \omega_i$
$K_f\ /\ K_i = (0.5\ I_f\ \omega_f^{\ 2})\ /\ (0.5\ I_i\ \omega_i^{\ 2}) = (2\ I_i\ *\ ((0.5)\ \omega_i)^{\ 2}$
$K_f\ /\ K_i = 0.5$

Question 13. A solid sphere and a hollow sphere have the same mass and radius. They are rotating with the same angular speed. Which one has the higher angular momentum?

a) the solid sphere
b) the hollow sphere
c) both have the same angular momentum
d) impossible to determine

$L=I\ \omega$
$I= (2/5)\ mr^2$ (solid)
$I= (2/3)\ mr^2$ (hollow)
Since (2/3) > (2/5) a HOLLOW sphere has a higher angular momentum than a solid sphere with the same mass, radius and angular velocity

Question 14. You are sitting on a stool mounted on a swivel stand. If you sit on stool with your arms stretched out, and we give the stool a spin. After that nobody else touches the stool but you bring your

arms in towards your body and wrap them around your body. What happens to the angular speed of the stool and why?
According to conservation of Angular Momentum,
Iω = constant
By wrap your hand around your body the moment of inertia will decrease so, your angular velocity will increase.

Problems

1. If the velocity of a 2.0 kg body changes from $\vec{v}_1 = (4.0\ \hat{\imath} + 5.0\ \hat{\jmath}) m/s$ to $\vec{v}_2 = (7.0\ \hat{\jmath})$ in a time interval of 3.0s

a) What is the average force that acts on the body?
b) What is the impulse delivered to the body?

(1) a)

$$\Delta \vec{p}_x = \vec{p}_{f_x} - \vec{p}_{i_x}$$
$$= m\vec{v}_{f_x} - m\vec{v}_{i_x}$$
$$= 0 - 2(4\hat{\imath})$$
$$= -8\hat{\imath} \text{ kg}\cdot\text{m/s}$$

$$\Delta \vec{p}_y = \vec{p}_{f_y} - \vec{p}_{i_y}$$
$$= m\vec{v}_{f_y} - m\vec{v}_{i_y}$$
$$= 2(7\hat{\jmath}) - 2(5\hat{\jmath})$$
$$= 4\hat{\jmath} \text{ kg}\cdot\text{m/s}$$

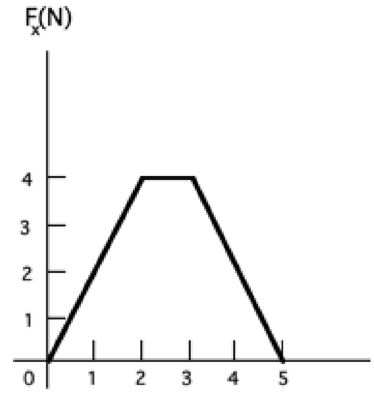
$$\Delta \vec{p}_{total} = \Delta \vec{p}_x + \Delta \vec{p}_y$$
$$= (-8\hat{\imath} + 4\hat{\jmath}) \text{ kg}\cdot\text{m/s}$$

$$\Delta \vec{p} = \Sigma \vec{F}_{av} \Delta t$$
$$\Sigma \vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t}$$
$$= \frac{-8\hat{\imath} + 4\hat{\jmath}}{3}$$
$$= (-2.67\hat{\imath} + 1.33\hat{\jmath}) \text{ N}$$

b)

$$\vec{J} = \Delta \vec{p}$$
$$= (-8\hat{\imath} + 4\hat{\jmath}) \underbrace{\text{N}\cdot\text{s}}_{\text{same as kg}\cdot\text{m/s}}$$

2. A horizontal force, F_x, acting on a 2.0 kg particle varies in time as shown in the figure to the right. Find
   a) the impulse of the force,
   b) the final velocity of the particle if it is initially at rest
   c) the final velocity of the particle if it is initially moving with a velocity of $(2.0\ \hat{\imath}) m/s$.



(2) a) $J = \text{AREA} = \triangle + \square + \triangle$

$= \frac{1}{2}(2)(4) + (1)(4)$

$= 12\ N \cdot s$

$\vec{J} = 12.0\ \hat{\imath}\ N \cdot s$

b) $\vec{J} = \Delta \vec{p}$

$\vec{p}_i = 0 \quad \therefore \quad \vec{p}_F = 12.0\ \hat{\imath}$

$m = 2\ kg \rightarrow \vec{v} = \frac{\vec{p}}{m} = 6.0$

c) $\vec{p}_i = (2)(-2\hat{\imath}) = -4\hat{\imath}\ kg\ m/s$

$\vec{p}_F = \vec{p}_i + \vec{J} = -4\hat{\imath} + 12\hat{\imath}$

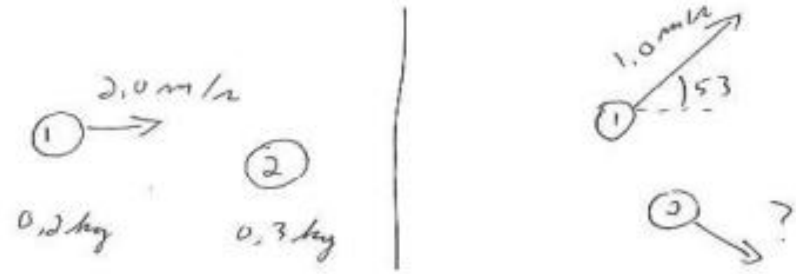
$= +8.0\ \hat{\imath}\ kg$

$\vec{v}_F = \frac{\vec{p}_F}{m} = +4.0\ \hat{\imath}\ m/s$

3. A 0.30 kg puck, initially at rest on a horizontal frictionless surface is struck by a 0.20 kg puck moving initially along the positive x-axis with a speed of 2.0 m/s. After the collision the 0.20 kg puck has a speed of 1.0 m/s at an angle $\theta = 53°$ to the positive x-axis.

a) Determine the velocity of the 0.30 kg puck after the collision.
b) Calculate the impulse experienced by each puck.
c) If the pucks were in contact for 0.0020 s, determine the average force exerted on the 0.20 kg puck.
d) Is the collision elastic? Explain.

(3) a)



x DIR

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad (m_2 v_2 \to 0)$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{0,2(2 - 1\cos 53)}{0,3} = 0,932 \text{ m/s}$$

y DIR

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \qquad (m_1 v_1 \to 0,\ m_2 v_2 \to 0)$$

$$v_2' = \frac{-m_1 v_1'}{m_2} = \frac{-0,2(1\sin 53)}{0,3} = -0,532 \text{ m/s}$$

$$\vec{v}_2' = (0,932\hat{i} - 0,532\hat{j}) \text{ m/s}$$

b) **0,3 kg PUCK (m_2)**

$$\vec{J} = \Delta \vec{p} = m \Delta \vec{v} \qquad \vec{v}_0 = 0$$

$$= 0,3\left[(0,432\hat{i} - 0,532\hat{j}) - 0\right]$$

$$= (0,280\hat{i} - 0,160\hat{j})\ N\cdot s$$

IMPULSE ON 0,2 kg PUCK : OPPOSITE OF THIS

$$\vec{J} = (-0,280\hat{i} + 0,160\hat{j})\ N\cdot s.$$

c) $\vec{J} = \vec{F}_{AV}\, \Delta t$

$$\vec{F}_{AV} = \frac{-0,280\hat{i} + 0,160\hat{j}}{.002}$$

$$= (-140\hat{i} + 80\hat{j})\ N$$

d) BEFORE

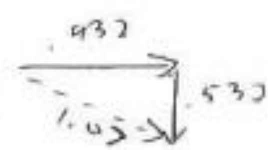
$m_1 \rightarrow 2\ m/s \qquad m_2 \rightarrow 0$

$$KE_{TOT} = \tfrac{1}{2}(0,2)(2)^2 + 0$$

$$= 0,40\ J$$

AFTER

$m_1 \rightarrow 1\ m/s \qquad m_2 \rightarrow 1,07\ m/s$



$$KE'_{TOT} = \tfrac{1}{2}(0,2)(1)^2 + \tfrac{1}{2}(0,3)(1,07)^2$$

$$= 0,272\ J$$

$KE'_{TOT} < KE_{TOT}$ $\therefore$ INELASTIC

4. A 3.0 kg mass moving in the positive *x* direction with a speed of 10 m/s collides with a 6.0 kg mass initially at rest. After the collision, the speed of the 3.0 kg mass is 8.0 m/s, and its velocity vector makes an angle of 35° with the positive *x*-axis. What is the magnitude of the velocity of the 6.0 kg mass after the collision?

(4)

x DIR

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad (m_2 v_2 \to 0)$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{3(10) - 3(8\cos 35)}{6}$$

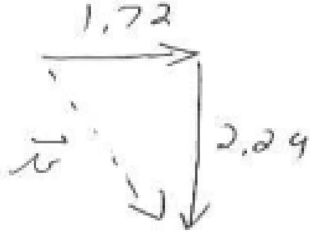
$$= 1.72 \text{ m/s}$$

y DIR

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad (m_1 v_1 \to 0,\ m_2 v_2 \to 0)$$

$$v_2' = \frac{-m_1 v_1'}{m_2} = \frac{-3(8\sin 35)}{6}$$

$$= -2.29 \text{ m/s}$$



$$|\vec{v}| = 2.86 \text{ m/s}$$

5. Two pucks, moving on a smooth horizontal surface, collide. Puck 1 has a mass 0.50 kg and an initial velocity of 2.0 m/s at 30°. Puck 2 has a mass of 0.30 kg and an initial velocity of 8.0 m/s at 300°. If, after the collision, Puck 1 moves with a velocity of 12.0 m/s at 350°, determine the final velocity of Puck 2.

(5)

x DIR

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 + m_2 v_2 - m_1 v_1'}{m_2}$$

$$= \frac{(0{,}5)(2\cos 30) + (0{,}3)(8\cos 60) - 0{,}5(12\cos 10)}{0{,}3}$$

$$= -12{,}8 \text{ m/s}$$

y DIR

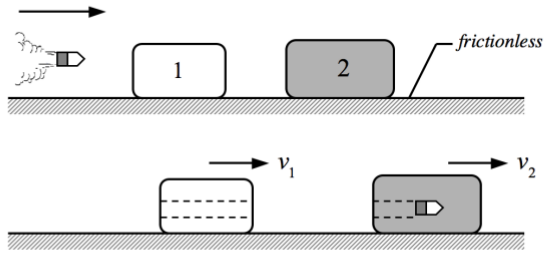
$$v_2' = \frac{0{,}5(2\sin 30) - 0{,}3(8\sin 60) - 0{,}5(-12\sin 10)}{0{,}3}$$

$$= -1{,}79 \text{ m/s}$$

$$\vec{v}_2' = (-12{,}8\,\hat{i} - 1{,}79\,\hat{j}) \text{ m/s}$$

NB KE' > KE! THERE MUST HAVE BEEN SOME SORT OF EXPLOSION WHEN THEY COLLIDED!!

6. A 3.50 g bullet is fired horizontally at two blocks, both at rest on a frictionless table. The bullet passes through block 1 (mass 1.20 kg) and embeds itself in block 2 (mass 1.80 kg). The blocks end up with speeds $v_1 = 0.630\,m/s$ and $v_2 = 1.40\,m/s$. Find the speed of the bullet as it (a) leaves block 1 and (b) enters block 1.



#6)

Collision with mass 2:

INITIAL FINAL

$$\Sigma P_i = \Sigma P_f$$

$$m_B v_B + m_2 v_2 = m_{B+2} v_{B+2}$$

$$(3.5E\text{-}3)(v_B) + (1.8)(0) = (3.5E\text{-}3 + 1.8)(1.4)$$

$$v_{B_i} = 721\ m/s$$

as it leaves m_1

Collision with mass 1:

INITIAL FINAL

$$\Sigma P_i = \Sigma P_f$$

$$m_{B_i} v_{B_i} + m_{1_i} v_{1_i} = m_{B_f} v_{B_f} + m_{1_f} v_{1_f}$$

$$(3.5E\text{-}3) v_{B_i} + 0 = (3.5E\text{-}3)(721) + (1.2)(0.63)$$

$$v_{B_i} = 937\ m/s$$

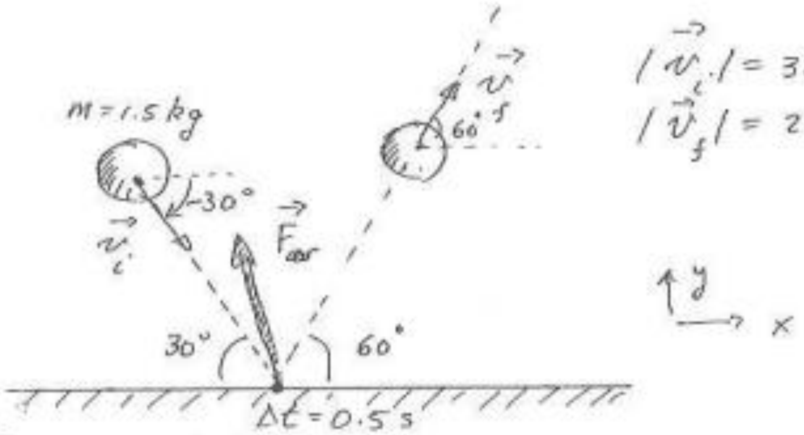
as it enters m_1

7. A 1.5 kg playground ball is moving with a velocity of 3.0 m/s directed 30° below the horizontal just before it strikes a horizontal surface. The ball leaves this surface 0.50 s later with a velocity of 2.0 m/s directed 60° above the horizontal. What is the magnitude of the average resultant force on the ball?

#7.

$|\vec{v}_i| = 3.0\,m/s$

$|\vec{v}_f| = 2.0\,m/s$



$\vec{P}_i = (3.9, -2.25)\ kg\,m/s$

$\vec{P}_f = (1.5, 2.60)\ kg\,m/s$

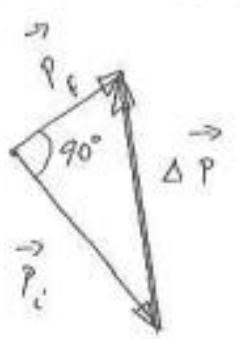
$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = (-2.4, 4.85)\ kg\,m/s$

$\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(-2.4, 4.85)}{0.5} = (-4.8, 9.70)\ N$

$\therefore\ |\vec{F}_{av}| = 10.8N$

direction = 116° (from x-axis)

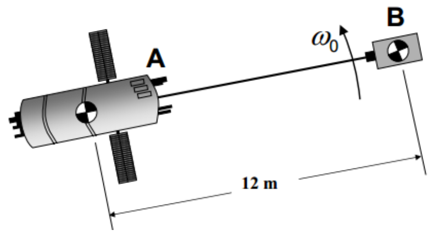
Using a vector diagram:



$\Rightarrow |\Delta\vec{P}| = \sqrt{4.5^2 + 3^2} = 5.41$

$\therefore\ \vec{F}_{av} = \frac{5.41}{\Delta t} = \frac{5.41}{0.5} = 10.8N$

8. Two research satellites ($m_A = 250\ kg$ and $m_B = 50\ kg$) are tied together by a 12.0 m long cable. The two satellites rotate about their center or mass with an angular speed of $\omega_o = 0.25\ rpm$ counterclockwise. If ground-control slowly extends the cable by an additional 6.0 m, how does the angular momentum of the satellites change?



The angular momentum is constant.

9. A solid, horizontal cylinder of mass 10.0 kg and radius 1.00 m rotates with an angular speed of 7.00 rad/s about a fixed vertical axis through its center. A 0.250-kg piece of putty is dropped vertically onto the cylinder at a point 0.900 m from the center of rotation and sticks to the cylinder. Determine the final angular speed of the system.

$I_i = I_1 = 0.5mr^2 = 0.5 \times 10 \times 1^2 = 5$ kg.m^2

$I_f = I_1 + I_2 = 5 + (0.250)(0.900)^2 = 5.2025$ kg.m^2

$L_i = L_f$

$I_i\ \omega_i = I_f\ \omega_f$

$5 \times 7 = 5.2025\ \omega_f$

$\omega_f = 6.73$ rad/s

10. The hour and minute hands of Big Ben in London are 2.79 m and 4.46 m long and have masses of 60.6 kg and 108 kg respectively.

a) Calculate the angular momentum of the minute hand about the center point of the clock. Treat the hand as long, thin rod, $I_{rod\ end} = \frac{1}{3}ML^2$.

$I_{rod\ end} = \frac{1}{3}ML^2 = (108)(4.46^2)/3 = 716.1$ kg.m^2

$\omega = \frac{2\pi}{60*60} = 1.74 \times 10^{-3}$ rad/s^2

$L = I\omega = (716.1)(1.74 \times 10^{-3}) = 1.25$ kg.m^2/s

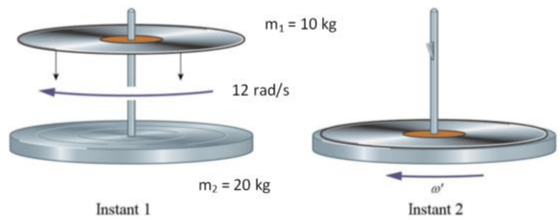
b) Calculate the angular momentum of the hour hand about the center point of the clock.

$I_{rod\ end} = \frac{1}{3}ML^2 = (108)(2.79^2)/3 = 157.2$ kg.m^2

$\omega = \frac{2\pi}{12*60*60} = 1.45 \times 10^{-4}$ rad/s^2

$L = I\omega = (157.2)(1.45 \times 10^{-4}) = 0.023$ kg.m^2/s

11. Two uniform disks of mass 10.0 kg and 20.0 kg respectively are free to rotate about a common frictionless axle. Each disk has a radius of 0.500 m and disk 1 is initially rotating at 12.0 rad/s while disk 2 is initially at rest. Disk 1 is then dropped onto disk 2 and after colliding the two disks rotate at a common angular velocity. Find that velocity.



$I_i = I_1 = 0.5mr^2 = 0.5 \times 10 \times (0.5)^2 = 1.25$ kg.m^2
$I_2 = 0.5mr^2 = 0.5 \times 20 \times (0.5)^2 = 2.5$ kg.m^2
$I_f = I_1 + I_2 = 1.25 + 2.5 = 3.75$ kg.m^2

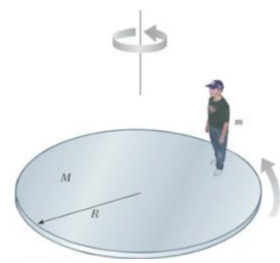
$L_i = L_f$

$I_i\, \omega_i = I_f\, \omega_f$

$1.25 \times 12 = 3.75\, \omega_f$

$\omega_f = 4$ rad/s

12. A 60-kg student stands on a rim of a horizontal platform of rotational inertia $I = 300\ kg \cdot m^2$ and radius 2-m. Initially the platform & student rotate together with angular velocity of 1.5 rad/s. The student walks slowly from the rim of platform toward the center.



a) What is the angular velocity of platform - student when the student is at 0.2 m from the center of platform?

$I_1 = I_{\text{student standing on the1 rim stands on the edge}} = m r_1^2 = (60)(2^2) = 240$ kgm^2
$I_2 = I_{\text{student is at 0.2 m from the center of platform}} = m r_1^2 = (60)(0.2^2) = 2.40$ kgm^2

$I_i = 300 + 240 = 540$ kg.m^2
$I_f = 300 + 2.40 = 302.4$ kg.m^2

$L_i = L_f$

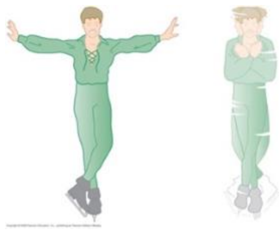
$I_i\, \omega_i = I_f\, \omega_f$

$(540)(1.5) = (302.4)\, \omega_f$

$\omega_f = 2.68$ rad/s

b) What is the change of kinetic energy of the system?

$\Delta KE = \frac{1}{2}(I_f \omega_f^2 - I_i \omega_i^2)$
$= \frac{1}{2}\ (302.4 * 2.68^2 - 540 * 1.5^2 = 478.48$J



13. The outstretched hands and arms of a figure skater preparing for a spin can be considered a slender rod pivoting about an axis through its center.
When the skater's hands and arms are brought in and wrapped around his body to execute the spin, the hands and arms can be considered a hoop. His hands and arms have a combined mass 8.0 kg.
When outstretched they span 1.8m; when wrapped, they form a hoop of radius 25 cm.
The moment of inertia about the rotation axis of the remainder of his body is constant and equal to 0.40 kgm^2.
If his original angular speed is 0.40 rev/s, what is his final angular speed?

When the arms are outstretched:

$I_{arms,\,i} = \frac{1}{12}\ mr^{2}$

$I_{arms,\,i} = \frac{1}{12}\ (8 * 1.8^{2}) = 2.16\ kg.m^2$

$I_i = I_{body} + I_{arms,\,i} = 0.4 + 12.96 = 2.56\ kg.m^2$

When the arms are wrapped:

$I_{arms,f} = m\,r^2$

$I_{arms,\,f} = 8 * 0.25^2 = 0.5\ kg.m^2$

$I_f = I_{body} + I_{arms,\,f} = 0.4 + 0.5 = 0.90\ kg.m^2$

$L_i = L_f$

$I_i\ \omega_i = I_f\ \omega_f$

$(2.56)\ (0.4) = (0.9)\ \omega_f$

$\omega_f = 1.14$ rev/s

note: hoop replaced ***thin walled hollow cylinder*** *in the question to align with Open Stax vocabulary*