SOLUTIONS PROBLEM SET # 5

Work; Energy; Power

Conceptual Questions

Question 1.
C is answer. Power is rate at which work is done. Same work, less time if greater power.

Question 2.
D is answer.
I and II are identical since they both slide without friction. III has enough friction to roll without slipping. This means that it will convert its potential energy into both rotational and linear kinetic energy. The **total** kinetic energy is still **mgh**, and hence the same as I and II (it will just be moving with less speed). IIII slides with friction, which means it loses energy to heat along the way from top to bottom. It will have less kinetic energy than I, II and III.

Question 3.

a) Elastic potential energy is $\frac{1}{2}kx^2$, not $\frac{1}{2}kx$ as shown.
You are not asked for y. You are instead asked for the distance along the incline, which is d, where $\frac{y}{d} = sin30$

b) $U_{spring} = U_{gravity}$

$$\frac{1}{2}kx^2 = mg(dsin30)$$

$$\frac{1}{2}(40\ 000)(0.50)^2 = (100)(9.8)(dsin30)$$

$$\rightarrow d = 10.2\ m$$

Question 4.
a) push a wagon moving to the left with your hand … to the left!
b) use less force in the situation above.
c) while the wagon rolls towards you, push it in the opposite direction to that motion with your hand.
d) the tension the string of a pendulum exerts on the 'bob' as it swings through an arc towards its lowest point.

Question 5.
C is answer. Use $P = \frac{mgy}{\Delta t}$

Question 6.

	True	False
the potential energy of the system increases	X	
the kinetic energy of the system decreases		X
the earth does negative work on the system	X	
you are doing a positive work	X	

Question 7.
3 is answer. Elastic potential energy has its lowest value at equilibrium and gets larger as the cart position increases away from equilibrium.

Question 8.

a) $KE_A > KE_B$. Same ω for both. A has larger I since mass is further from rotation axis, where $KE = \frac{1}{2}I\omega^2$

b) $KE_A = KE_C$. Both have same ω and same $I = mR^2$.

c) $KE_A > KE_D$. Same ω for both. A has larger I since mass is further from rotation axis, where $KE = \frac{1}{2}I\omega^2$

d) $KE_A > KE_E$. Same ω for both. A has larger I ($I_A = mR^2$ vs $I_B = \frac{1}{2}mR^2$), where $KE = \frac{1}{2}I\omega^2$

e) **A** and **C** have largest KE. Same ω for all. A and C have largest I.

f) B has smallest KE. Same ω for all. B has smallest I with its mass concentrated closest to rotation axis.

g) B<E<D<A=C. See above for A, B, C reasoning. For placement of D relative to E, $I_D = \frac{m}{2}\left(\frac{R}{2}\right)^2 + \frac{m}{2}(R)^2 = \frac{5mR^2}{8}$ which is larger than $I_E = \frac{1}{2}mR^2$

Problems

Problem 1

a) Work done $= \int F\,dx$ = Area under F vs x curve.

$W_{0\to 2m} = 3.2\,J$

b) $W_{net} = \Delta K$ $\quad\quad \Delta K = K_f - K_i$

$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$K_f - \frac{1}{2}(3)(1.5)^2 = 3.2\,J$

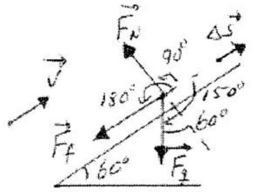
$K_f = 6.58\,J$

c) $K_f = \frac{1}{2}mv_f^2$ $\quad\quad v_f = \sqrt{\frac{2(6.58)}{3}} = 2.09\,m/s$

d) $W_{net} = 5\,J$

e) $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3)(1.5)^2 = 3.4\,J$

Problem 2



$m = 2\,kg$
$V_i = 3\ m/s$
$V_f = 0$
$\mu = 0.3$
$\Delta S = ?$
$W = F(\cos\theta)\Delta S$

(a) F_N = Normal force
F_g = Weight = $mg = \underline{20}$
F_f = Friction
$= (\mu mg \cos 60^\circ)$
$= (0.3)(20)\cos 60^\circ$
$= \underline{3N}$

$\boxed{F_f = 3N}$

(b) $W_N = F_N(\cos 90^\circ)\Delta S = \underline{0}$
$W_g = mg(\cos 150^\circ)\Delta S = 20(\cos 150^\circ)\Delta S = \underline{(-17.32)\Delta S}$
$W_f = F_f(\cos 180^\circ)\Delta S = 3(\cos 180^\circ)\Delta S = \underline{(-3)\Delta S}$

$\boxed{W_T = \Delta K}$

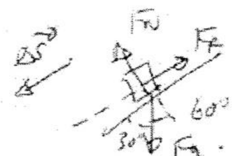
$W_g + W_N + W_f = \frac{1}{2}m(V_f^2 - V_i^2)$

$(-17.32\,\Delta S) + 0 + (-3.00\,\Delta S) = \frac{1}{2}(2)(0-9)$

$\therefore\ \Delta S = \dfrac{-9}{-20.32} = \boxed{0.443\,m}$ up the plane.

(a)(cont.) $W_g = (-17.32)\Delta S = -17.32\,(0.443) = -\underline{7.67J}$
$W_f = (-3)\Delta S = -3(0.443) = -\underline{1.33J}$

(c) Down incline:



$V_i = 0$

$\boxed{W_T = \Delta K}$

$W_g + W_N + W_f = \frac{1}{2}m(V_f^2 - V_i^2)$

where:

$\Delta S = 0.443\,m$ (from (b))

$W_g = mg(\cos 30^\circ)\Delta S$
$= 20(\cos 30^\circ)0.443$
$= \underline{7.67J}$

$W_f = -(\mu mg \cos 60^\circ)\Delta S$
$= -(.3)(20)\cos 60^\circ(0.443)$
$= -\underline{1.329}\,J$

$W_N = \underline{0}$

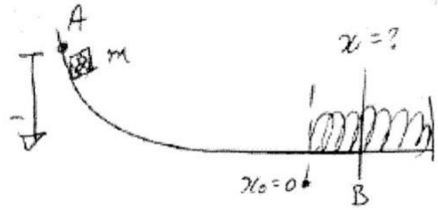
(d) $\boxed{W_T = \Delta K}$

$7.67 - 1.33 = \frac{1}{2}(2)(V_f^2 - 0)$

$V_f = \sqrt{6.34}$

$= \boxed{2.52\,m/s}$, at initial position

Problem 3



$V_A = V_B = 0$

$\Delta y = -5\,m$

$m = 3\,kg$

$k = 400\,N/m$

(a) $\boxed{W_T = \Delta K}$ ($A \to B$, on block)

$$W_g + W_s = \tfrac{1}{2} m (V_B^2 - V_A^2)$$

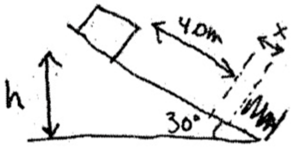
$$\left(-mg\Delta y - \tfrac{1}{2} k x^2\right) = 0$$

$$(-30)(-5) - \tfrac{1}{2}(400)\,x^2 = 0$$

$$x = \sqrt{\frac{150}{200}} = \boxed{0.866\,m}$$

(b) After coming to rest at B, the block will be projected to the left by the spring and it will continue along and up the ramp to its initial position, because there is no friction.

Problem 4



a) surface is frictionless

$$E_{initial} = E_{final}$$

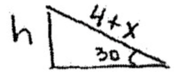
$$U_g + K + U_s = U_g + K + U_s$$

$$mgh + \tfrac{1}{2}mv^2 + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

"released" implies 0

choose 0 at bottom

since block moves a total distance of (4+x) down the incline, we have.



$$h = (4+x)\sin 30$$

$$mg(4+x)\sin 30 + 0 + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

$$2(9.8)(4+x)\tfrac{1}{2} = \tfrac{1}{2}(100)x^2$$

$$39.2 + 9.8x - 50x^2 = 0$$

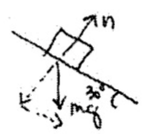
$$x = \frac{-9.8 \pm \sqrt{(9.8)^2 - 4(-50)(39.2)}}{2(-50)}$$

$$x = \frac{-9.8 \pm 89.1}{-100} = 0.989\,m \text{ or } -0.793\,m$$

b) with friction along incline

$$E_{initial} - W_{friction} = E_{final}$$

$$U_g + K + U_s - F_f d = U_g + K + U_s$$



$$F_{net} = 0$$

$$n - mg\cos 30 = 0$$

$$n = mg\cos 30$$

$$mg(4+x)\sin 30 + 0 + 0 - \mu(mg\cos 30)(4+x) = 0 + 0 + \tfrac{1}{2}kx^2$$

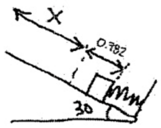
$$2(9.8)(4+x)\tfrac{1}{2} - 0.2(2)(9.8)(0.866)(4+x) = \tfrac{1}{2}(100)x^2$$

$$39.2 + 9.8x - 13.58 - 3.39x - 50x^2 = 0$$

$$25.6 + 6.4x - 50x^2 = 0$$

$$x = \frac{-6.4 \pm \sqrt{(6.4)^2 - 4(-50)(25.6)}}{2(-50)} = \frac{-6.4 \pm 71.8}{-100} = 0.782\,m \text{ or } -0.654\,m$$

c) how far up incline ie. what distance until it stops.



$$E_{initial} - W_f = E_{final}.$$

$$U_g + K + U_s - f_f d = U_g + K + U_s.$$

$$0 + 0 + \tfrac{1}{2}(100)(0.782)^2 - \mu mg\cos 30(x+0.782) = mg(x+0.782)\sin 30 + 0 + 0$$

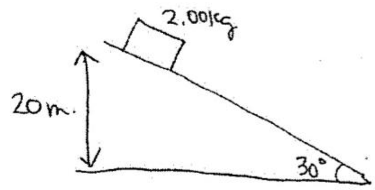
$$30.6 - 0.2(2)(9.8)(0.866)(x+0.782) = 2(9.8)(x+0.782)\tfrac{1}{2}$$

$$30.6 - 3.39x - 2.65 = 9.8x + 7.66$$

$$20.29 = 13.19x$$

$$\boxed{x = 1.54\text{m}}$$

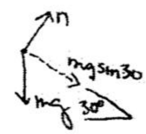
Problem 5



a) $U_g = mgh$

$= 2(9.8)(20) = 392\text{J}$

b) $F_{net} = ma$



$mg\sin 30 = ma$

$a = 9.8\sin 30$

$a = 4.9\text{m/s}^2$ down incline

$V = V_0 + at$

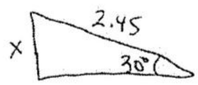
$V = 0 + 4.9(1) = 4.9\text{m/s}.$

starts from rest

c) at 1s, the box has slid a distance down the incline

$\Delta X = V_0 t + \frac{1}{2}at^2$

$\Delta X = 0 + \frac{1}{2}(4.9)(1)^2 = 2.45\text{m}$



this corresponds to a height

$X = 2.45\sin 30 = 1.225\text{m}$

(down from the 20m it was initially above the ground)

$U_g = mg(20 - 1.225) = 2(9.8)(18.775) = 368\text{J}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(2)(4.9)^2 = 24\text{J}$ (total is still 392J)

d) at the bottom

$E_{initial} = E_{final}$

$U_g + K = U_g + K$

$mgh + 0 = 0 + \frac{1}{2}mV^2$

$2(9.8)(20) = \frac{1}{2}(2)V^2$

$V = 19.8\text{m/s}.$

the kinetic energy should be equal to the potential energy at the top, since all of it is transferred (no loss due to friction etc...)

Problem 6

$$K_{total} = K_{translational} + K_{rotational}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}m\cancel{r^2}\left(\frac{v^2}{\cancel{r^2}}\right)$$

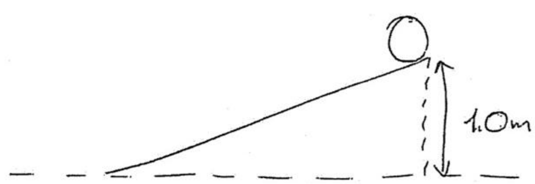
$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{3}{4}mv^2$$

$$= \frac{3}{4}(10)(5)^2$$

$$= 187.5 \text{ J}$$

Problem 7



a) frictionless ramp.

$$U_i + \cancel{K_i}^{\,0} + \cancel{W_{nc}}^{\,0} = \cancel{U_f}^{\,0} + K_f$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \cdot 9.81 \cdot 1} = \underline{\underline{4.4}}\ \frac{m}{s}$$

b) ramp with enough friction for rolling without slipping.

$$U_i + \cancel{K_i} + \cancel{W_{nc}} = \cancel{U_f} + K_f$$

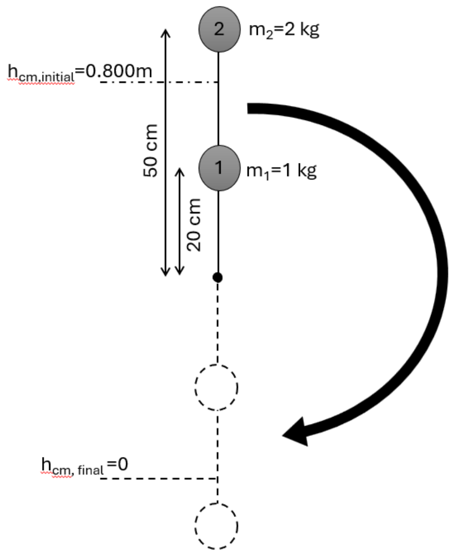
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$mgh = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4}{3} \cdot g \cdot h} = \sqrt{\frac{4}{3} \cdot 9.81} = \underline{\underline{3.62}}\ \frac{m}{s}$$



Problem 8

a) Moment of inertia of the pendulum

$$I = m_1 \cdot r_1^2 + m_2 \cdot r_2^2 = 1 \cdot 0.2^2 + 2 \cdot 0.5^2$$
$$= 0.54\ kg \cdot m^2$$

b) Position of centre of mass from pivot point

$$y_{cm} = \frac{y_1 \cdot m_1 + y_2 \cdot m_2}{m_1 + m_2} = \frac{0.2 \cdot 1 + 0.5 \cdot 2}{1 + 2} = 0.40\ m$$

The position of the centre of mass therefore changes by 0.800 m from the initial to the final position.

c)

$$U_i + K_i + W_{nc} = U_f + K_f \to U_i = K_f$$

$$mgh_{cm} = \frac{1}{2} I\omega^2 \to \omega = \sqrt{\frac{2 \cdot mgh_{cm}}{I}}$$

$$= \sqrt{\frac{2 \cdot 3 \cdot 9.81 \cdot 0.800}{0.54}} = 9.34 \frac{rad}{s}$$

d) Both spheres have the same angular velocity, but the 1 kg sphere moves at v=ωr=9.34*0.2=1.87 m/s and the 2 kg sphere moves at v=ωr=9.34*0.5=4.67 m/s

Problem 9

known values:

$$m = 1.10 \times 10^3 kg$$
$$a = 4.60\ m/s^2$$
$$t = 5.00s$$
$$v_o = 0\ m/s$$
$$x_o = 0\ m$$

$$v = v_o + at$$
$$v = 0 + 4.6(5) = 23\ m/s$$

$$x = x_o + v_0 t + \frac{1}{2} at^2$$
$$x = 0 + 0 + 0.5(4.6)(5)^2 = 57.5\ m$$

$$F = ma = 1.10 \times 10^3 (4.6) = 5060\ N$$

$$P = F \cdot v_{avg} = 5060 \cdot \frac{23+0}{2} = 5.82\ kW \qquad \textbf{OR} \qquad P = \frac{W}{t} = \frac{F \cdot d}{t} = \frac{5060(57.5)}{5} = 5.82\ kW$$

Problem 10

a) known values:

$$m = 1800\ kg$$
$$F_g = 1800(9.8) = 17640\ N$$
$$F_f = 4000\ N$$

$$F_{net} = ma$$
$$F_{motor} - F_g - F_f = 0$$
$$F_{motor} = 21640\ N$$
$$P = F_{motor} \cdot v = 21640(3) = 64900\ \frac{J}{s} = 64.9\ kW$$

b)

$$F_{net} = ma$$
$$F_{motor} - F_g - F_f = 1800(1.3)$$
$$F_{motor} = 21640 + 2340 = 23980$$
$$v = v_o + at$$
$$v = 0 + 1.3(4) = 5.2\ m/s$$
$$P = F_{motor} \cdot v = (23980 \cdot 5.2) = 125\ kW$$

Problem 11

a) $\tau = I\alpha$

$$10 = 2\alpha \rightarrow \alpha = 5.0\frac{rad}{s^2}$$

known values:

$$\omega_o = 0.0\ \frac{rad}{s}$$
$$t = 8.0\ s$$
$$\omega = \omega_o + \alpha t$$
$$\omega = 0 + 5(8) = 40\frac{rad}{s}$$
$$K = \frac{1}{2}I\omega^2 = 0.5(2)(40)^2 = 1600\ Joules$$

$$W_{total} = \Delta K = K_{final} - K_{initial} = 1600 - 0 = 1600\ Joules.$$

OR

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$\theta = 0 + 0 + \tfrac{1}{2}(5)8^2 = 160\ radians$$

$$W_{total} = Fd = F(r\theta) = \tau\theta = 10(160) = 1600\ Joules.$$

b) $P_{avg} = \frac{W}{t} = \frac{1600}{8} = 200\ W$