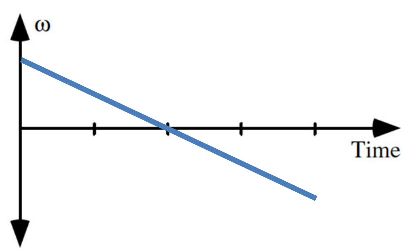
# SOLUTIONS: PROBLEM SET #2
# CIRCULAR and ROTATIONAL MOTION with CONSTANT ACCELERATION

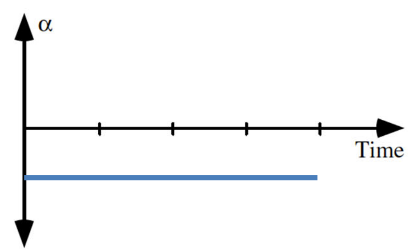
**Question 1.**

Ans: c) We know that $a = \frac{v^2}{r}$ and since the magnitude of the acceleration is constant, the speed of the planet is also constant. The planet travels a distance = $2\pi r$ in a time $T$ giving $v = \frac{2\pi r}{T}$. Substitute and rearrange to show that $T^2 = \frac{4\pi^2 r}{a}$.

**Question2.**

Since the torque opposes the initial counterclockwise rotation, the angular acceleration is directed clockwise (therefore negative).




**Question 3.**

a) The disk is initially rotating at a constant positive angular velocity. After approximately 2.2 s has passed, the disk's angular velocity decreases suddenly and continues rotating at a constant velocity in the same direction as it was initially rotating for an additional 2.2 s.

b) The disk experienced a sudden force which was applied briefly causing it to slow down. For example, something may have bumped into the disk.

**Question 4.**

a) The graph on the left is parabolic in shape suggesting that the angular acceleration is constant. The vertical intercept of the graph on the left tells us that the initial angular position of the disk is zero, $\theta_o = 0$. The initial slope of the graph on the left indicates that the disk is initially at rest (initial angular velocity is zero), $\omega_o = 0$. If the disk is indeed rotating with constant angular acceleration, then $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2$ which means the slope of the graph on the right would be constant (equal to $\frac{1}{2}\alpha$). The graph on the right is indeed linear which indicates constant angular acceleration.

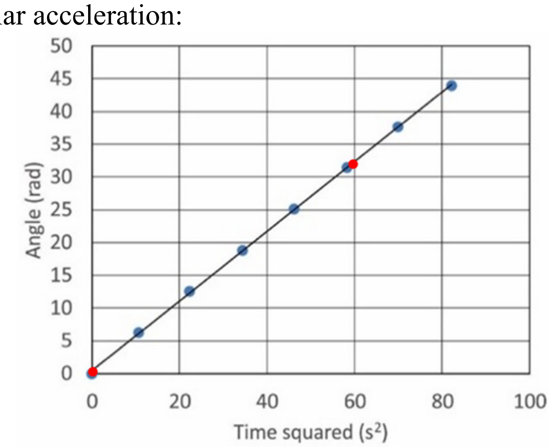
b) Using the slope of the graph on the right, we can solve for the angular acceleration:

$$slope = \frac{1}{2}\alpha$$

$$\frac{\Delta\theta}{\Delta(t^2)} = \frac{1}{2}\alpha$$

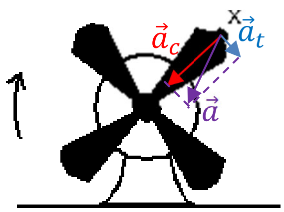
$$\frac{32.5 - 0}{60 - 0} = \frac{1}{2}\alpha$$

$$\alpha = \boxed{1.08\, rad/s^2}$$



**Question 5.**
Ans: c)

**Question 6.**
Ans: b) The total acceleration is the vector sum of the centripetal acceleration and the tangential acceleration. The centripetal acceleration is directed towards the centre of rotation and the tangential acceleration is in the same direction as the instantaneous velocity of point X (since the fan is speeding up).




**Question 7.**
Since the teeth are equally spaced and well-meshed, the tangential speed of the teeth on the small gear will equal the tangential speed of the teeth on the large gear:

$$v_{t\ small} = v_{t\ large}$$
$$v_{t\ small} = R_{large}\omega_{large}$$

Therefore, the speed of the teeth of the small gear depends on the radius of the large gear (since $\omega_{large}$ is constant)

$$v_{small\ A} = 0.4\omega_{large}$$
$$v_{small\ B} = 0.4\omega_{large}$$
$$v_{small\ C} = 0.8\omega_{large}$$
$$v_{small\ D} = 0.6\omega_{large}$$
$$v_{small\ E} = 0.2\omega_{large}$$
$$v_{small\ F} = 0.6\omega_{large}$$
$$\boxed{v_{small\ E} < v_{small\ A} = v_{small\ B} < v_{small\ D} = v_{small\ F} < v_{small\ C}}$$

**Question 8.**
Since the belt is no-slip, the speed of the belt will equal the tangential speed of all points on the outer rim of the small (and large) wheel:

$$v_{belt} = v_{t\ small} = R_{small}\omega_{small}$$

Therefore, the speed of the belt depends on the radius of the small gear (since $\omega_{small}$ is constant)

$$v_{belt\ A} = 0.2\omega_{small}$$
$$v_{belt\ B} = 0.3\omega_{small}$$
$$v_{belt\ C} = 0.4\omega_{small}$$
$$v_{belt\ D} = 0.5\omega_{small}$$
$$v_{belt\ E} = 0.1\omega_{small}$$
$$v_{belt\ F} = 0.2\omega_{small}$$
$$\boxed{v_{belt\ E} < v_{belt\ A} = v_{belt\ F} < v_{belt\ B} < v_{belt\ C} < v_{belt\ D}}$$

**Question9.**
a) Yes. For example, consider a car moving along a straight road. As the car speeds up or slows down, it experiences linear acceleration. Since the car doesn't turn (i.e., no rotation), its angular acceleration remains zero.
b) Yes. For example, consider a bug sitting on the very centre of a spinning fan. As the fan spins faster or slower, the bug experiences angular acceleration. Since the bug remains stationary in terms of linear motion (no translational movement), its linear acceleration is zero.

# PROBLEMS

1. $\omega_o = 33\ rev/min,\ \omega = 0,\ t = 2\ min$

a) $\omega = \omega_o + \alpha t$

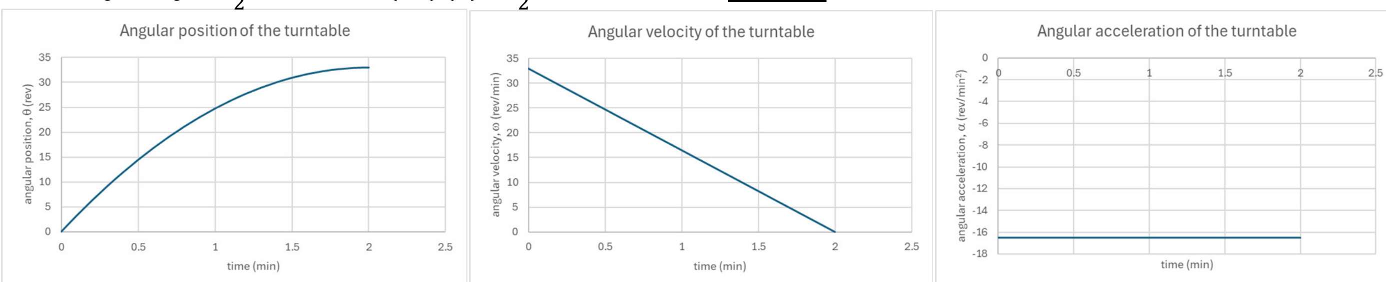
$33 = 0 + \alpha(2)$

$\alpha = \boxed{-16.5\ rev/min^2}$

b) since the angular acceleration is constant, $\omega_{ave} = \frac{\omega_o+\omega}{2} = \frac{33+0}{2} = \boxed{16.5\ rev/min}$

c) $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 = 0 + (33)(2) + \frac{1}{2}(-16.5)(2^2) = \boxed{33\ rev}$

d)




2. $r = 10\ cm = 0.1\ m,\ \omega_o = 0,\ \alpha = 10\ rad/s^2,\ t = 5s$

a) $\omega = \omega_o + \alpha t = 0 + (10)(5) = \boxed{50\ rad/s}$

b) $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 = 0 + (0)(5) + \frac{1}{2}(10)(5^2) = 125\ rad \cdot \frac{1\ rev}{2\pi\ rad} = \boxed{19.9\ rev}$

3. $\omega_o = 500\ rev/min,\ \omega = 3000\ rev/min,\ t = 5\ s$

a)

$$\omega_o = 500\frac{rev}{min}\cdot\frac{2\pi\ rad}{1\ rev}\cdot\frac{1\ min}{60\ s} = \boxed{52.4\ rad/s}$$

$$\omega = 3000\frac{rev}{min}\cdot\frac{2\pi\ rad}{1\ rev}\cdot\frac{1\ min}{60\ s} = \boxed{314\ rad/s}$$

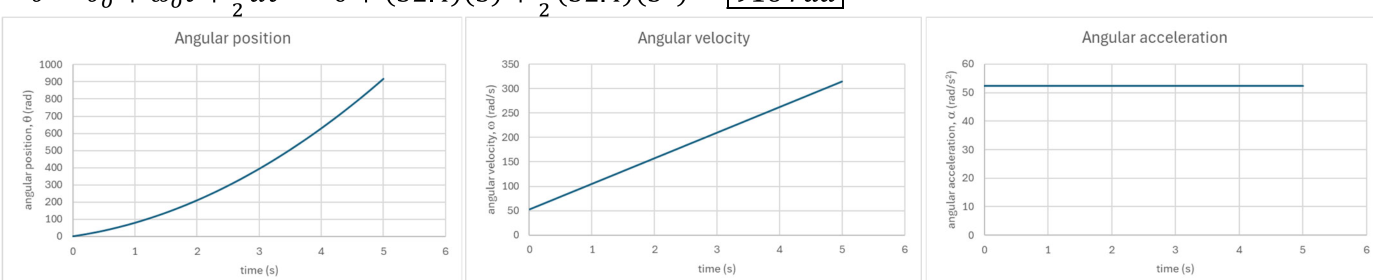
b) $\omega = \omega_o + \alpha t$

$314 = 52.4 + \alpha(5)$

$\alpha = \boxed{52.4\ rad/s^2}$

c) $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 = 0 + (52.4)(5) + \frac{1}{2}(52.4)(5^2) = \boxed{916\ rad}$

d)




4. $\omega_o = 0,\ t = 10\ s,\ \theta = 5\ rev = 10\pi\ rad$

a) $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$$10\pi = 0 + (0)(10) + \frac{1}{2}(\alpha)(10^2)$$

$\alpha = \boxed{0.628\ rad/s^2}$

b) $\omega = \omega_o + \alpha t = 0 + (0.628)(10) = \boxed{6.28\, rad/s}$

c) $r = 0.36\, m$

$\Delta s = r\Delta\theta = (0.36)(10\pi) = \boxed{11.3\, m}$

5. $v_o = 90 km/h$, $v = 50\ km/h$, $t = 15\ s$, $r = 150\ m$

a) Convert km/h to m/s:

$$v_o = 90km/h = \frac{90 \cdot 1000m}{3600s} = 25.0\ m/s$$

$$v = 50km/h = 13.9m/s$$

The tangential acceleration: $a_t = \frac{\Delta v}{\Delta t} = \frac{13.9-25.0}{15} = -0.74\ m/s^2$

The magnitude of the centripetal (radial) acceleration when $v$ = 50km/h is: $a_c = \frac{v^2}{R} = \frac{13.9}{150} = 1.29 m/s^2$

The total acceleration is given by: $\vec{a} = \vec{a}_c + \vec{a}_t$

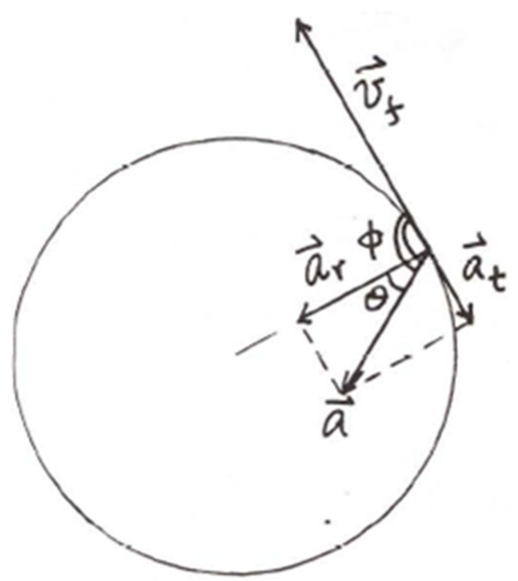
The magnitude of the total acceleration is: $a = \sqrt{a_c^2 + a_t^2}$

When $v$ = 50 km/h, $a = \sqrt{1.29^2 + 0.74^2} = \boxed{1.49 m/s^2}$

at $\theta = cos^{-1} = \frac{a_c}{a} = 30.0^\circ$

The angle between $\vec{v}_f$ and $\vec{a}$ is $\phi = 90 + \theta = \boxed{120^\circ}$

b)



6.

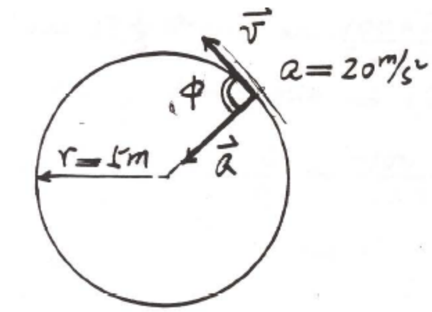
$$a_t = \frac{dv}{dt} = a\cos\phi$$

and $$a_r = a\sin\phi = \frac{v^2}{r}$$

$$v = \sqrt{ar\sin\phi}$$

a) $a_t = 20 \cdot \cos 90 = \boxed{0 m/s^2}$

$v = \sqrt{20 \cdot 5 \cdot \sin 90} = \boxed{10 m/s}$

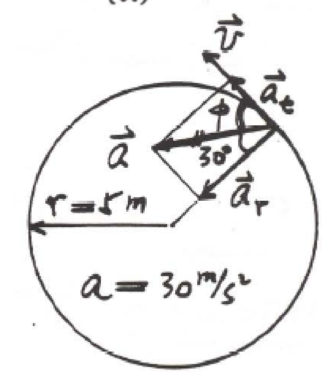


(a)

b) $\phi = 90 - 30 = 60^\circ$

$a_t = 30\cos 60 = \boxed{15 m/s^2}$

$v = \sqrt{30 \cdot 5 \cdot \sin 60} = \boxed{11.4 m/s}$

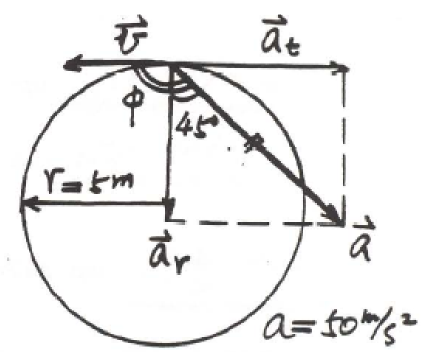


(b)

c) $\phi = 90 + 45 = 135^\circ$

$a_t = 50 \cdot \cos 135 = \boxed{-35.4 m/s^2}$

$v = \sqrt{50 \cdot 5 \cdot \sin 135} = \boxed{13.3 m/s}$



(c)

7.

$a_t = 5.0 m/s^2$

$a_r = \frac{v^2}{r} = \frac{3^2}{2} = 4.5 m/s^2$

$a_{tot} = \sqrt{a_r^2 + a_t^2} = \sqrt{5^2 + 4.5^2} = \boxed{6.73 m/s^2}$

8.

$r_1 = 0.6\,m$ , $\omega_1 = 8$ revs/s $= 16\pi$ rad/s $= 50.3$ rad/s

$r_2 = 0.9\,m$ , $\omega_2 = 6$ rev/s $= 12\pi$ rad/s $= 37.7$ rad/s

$V_1 = \omega_1 r_1 = (50.3 \text{ rad/s})(0.6 m) = \underline{\underline{30.2 \text{ m/s}}}$

$V_2 = \omega_2 r_2 = (37.7 \text{ rad/s})(0.9 m) = \underline{\underline{33.6 \text{ m/s}}}$

$\therefore V_2 > V_1$ , so $\underline{\underline{6 \text{ rev/s}}}$ gives the larger linear speed.

9.

$r = 10$ cm

$\omega_i = 0$

$\alpha = 10$ rad/s²

$\Delta t = 5s$

(a) $\omega_f = \omega_i + \alpha \Delta t$

$= 0 + 10\,(5)$

$= \underline{\underline{50 \text{ rad/s}}}$

(b) $a_t = \alpha r$

$= (10 \text{ rad/s}^2)(10 \text{ cm})$

$= \underline{\underline{100 \text{ cm/s}^2}} = 1 \text{ m/s}^2$

Linear speed $= V_f = \omega_f r$

$= (50 \text{ rad/s})(10 \text{ cm})$

$= \underline{\underline{500}} \text{ cm/s}^2 = 5 \text{ m/s}^2$

$\therefore$ Linear velocity is 5 m/s² directed along a tangent.

10.

$r = 0.2\ m$

$\omega_i = 0$

$\alpha = 2.0\ rad/s^2$

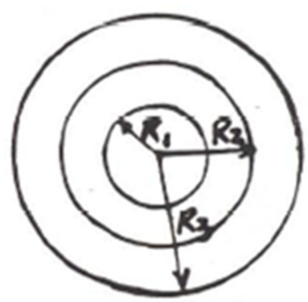
$\omega_f = 1.2\ rad/s$

At all times, $a_t = \alpha r = (0.2)(2) = \underline{0.40\ m/s^2}$

At required $t$; $a_r = \omega_f^2 r = (1.2)^2(0.2) = \underline{0.288\ m/s^2}$

Magnitude of $a_{tot.} = \sqrt{a_t^2 + a_r^2} = \sqrt{(0.4)^2 + (0.288)^2} = \underline{0.49\ m/s^2}$

11. $\omega_o = 1800 rev/\min = 60\pi rad/s$




a) all pulleys are on the same shaft

$v_1 = R_1\omega_o = 5cm \cdot 60\pi rad/s = \boxed{942\ cm/s}$

$v_2 = R_2\omega_o = 10cm \cdot 60\pi rad/s = \boxed{1.89 \times 10^3\ cm/s}$

$v_3 = R_3\omega_o = 15cm \cdot 60\pi rad/s = \boxed{2.83 \times 10^3\ cm/s}$

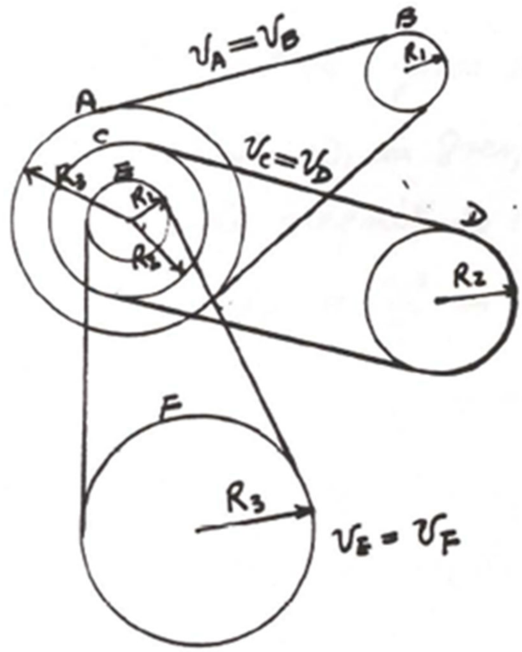
b)

i) belt connecting A and B:

$v_A = v_B$

$\omega_o R_3 = \omega_1 R_1$

$\omega_1 = \frac{R_3}{R_1}\omega_o = \frac{15cm}{5cm}1800rev/\min = \boxed{5400rev/\min}$




ii) belt connecting C and D;

$v_C = v_D$

$\omega_o R_2 = \omega_2 R_2$

$\omega_2 = \omega_o = \boxed{1800rev/\min}$

iii) belt connecting E and F:

$v_E = v_F$

$\omega_o R_1 = \omega_3 R_3$

$\omega_3 = \frac{R_1}{R_3}\omega_o = \frac{5cm}{15cm}1800rev/\min = \boxed{600rev/\min}$

12.

a)

$\omega_i = 500(rev/\min) = 500\frac{2\pi\ rad}{60s} = 52.36rad/s$

$\omega_f = 3000(rev/\min) = 3000\frac{2\pi\ rad}{60s} = 314.16rad/s$

$\Delta t = 5s$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{314.16 - 52.36}{5} = \boxed{52.4\ rad/s^2}$

b)

$\omega_f = 314.16rad/s$

$r = \frac{0.5m}{2} = 0.25m$

$v_f = \omega_f \cdot r = 314.16 \cdot 0.25 = \boxed{78.5\ m/s}$

c)

$r = \frac{0.5m}{2} = 0.25m$

$\alpha = 52.36rad/s^2$

$a_t = \alpha r = 52.36rad/s^2 \cdot 0.25m = \boxed{13.1\ m/s^2}$

d)

$$r = \frac{0.5m}{2} = 0.25m$$

$$\omega_f = 314.16 rad/s$$

$$a_r = \frac{v_f^2}{r} = \frac{(\omega_f r)^2}{r} = \omega_f^2 r = (314.16 rad/s)^2 \cdot 0.25m = \boxed{2.47 \times 10^4\ m/s^2}$$

e)

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(24674)^2 + 13.04^2} = \boxed{2.47 \times 10^4\ m/s^2}$$

$$tan\,\theta = \frac{a_t}{a_r} = \frac{13.04}{24674} \rightarrow \theta = \boxed{0.03°}$$

f)

first find $\Delta\theta$ in radians, then convert to deg

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$$

$$\Delta\theta = 52.36 rad/s \cdot 5s + \frac{1}{2} \cdot 52.36 rad/s^2 \cdot (5)^2$$

$$\Delta\theta = 916.3 rads$$

$$\Delta\theta = 916.3 \cdot \frac{360°}{2\pi\, rad} = \boxed{5.25 \times 10^4\ °}$$

(g) $\Delta\theta = 916$ rads

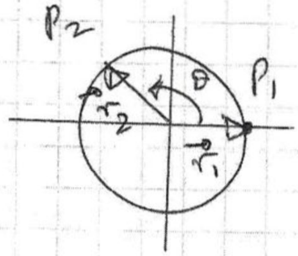
$r = 0,25$ m

$\Delta\theta = r\,\Delta\theta$

$= (0,25)(916)$

$= 229$ m

(h) Assuming the point starts on the x-axis, and $P_1$ is at (0,25 m; 0),




then $\vec{r}_1$ = 0,25 m at 0°.

To find $\vec{r}_2$ direction, $\theta$:

$$\Delta\theta = [(n \text{ revs}) + \theta]$$

Dividing $\Delta\theta$ by 360° gives "n" with remainder of "$\theta$".

$$\frac{52500}{360} \Rightarrow 145 \text{ rotations and } 0{,}833 \text{ revs.}$$

so $\theta = 0{,}833 \times 360° = \underline{300°}$

Position in polar form $\vec{r}_2$ = <u>0,25 m at 300°</u>