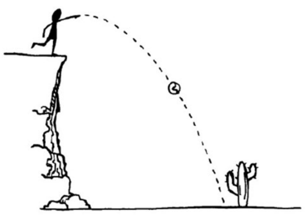
Solution for PROBLEM SET # 1
Motion in 1 and 2 Dimension

CONCEPTUAL QUESTIONS

Question 1.



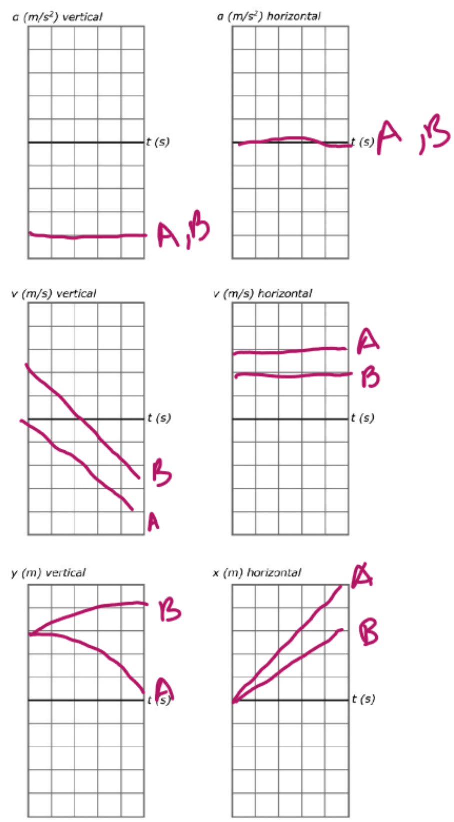
Scenario

A rock is thrown horizontally with velocity $\vec{v} = v_x \hat{\imath}$ from the top of a cliff of height H, as shown.

Using representations

Part A: sketch the following graphs of the motion of the rock on the axes provided.

See graphs below



Part B: A second rock is now thrown at an angle θ above the horizontal at the same speed $|\vec{v}|$ and from the same height H as the rock in Part A. On the same set of axes, sketch the graphs for the second rock. If the graphs for Part B are different than those for Part A, use a different colours or different lines to differentiate between the two situations.

Argumentation

Part C: If the second rock was instead thrown horizontally with initial speed $2|\vec{v}|$, would the horizontal distance *D* between the bottom of the cliff and the place where the rock lands be larger, smaller, or the same as the first rock? Check the appropriate blank for the claim and fill in the blanks of evidence.

Claim:

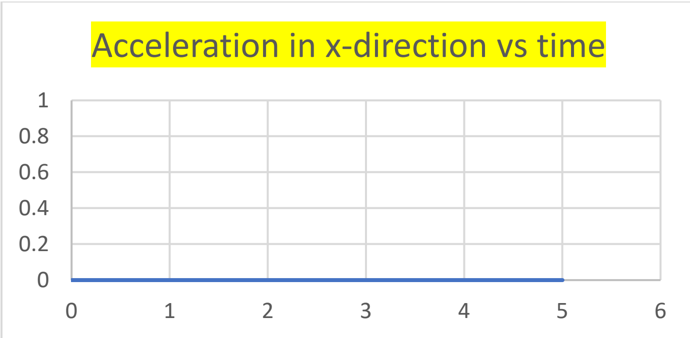
The horizontal distance between the bottom of the cliff and the place where the rock land is…

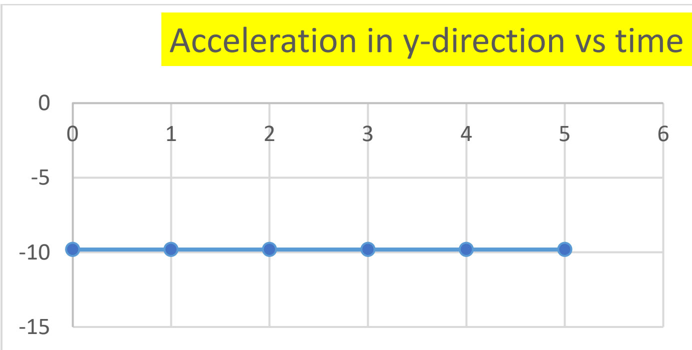
___ Larger ____ Smaller ___ The same

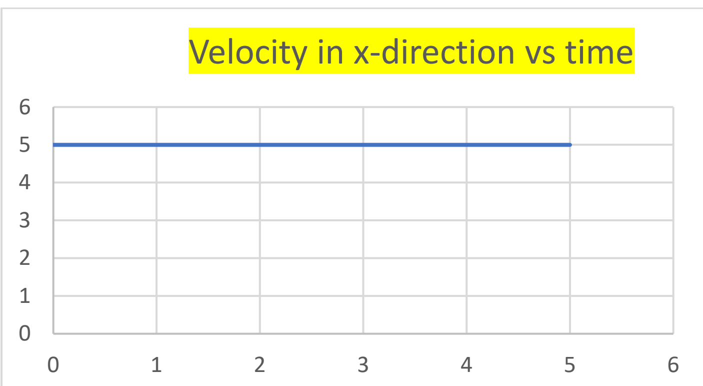
Evidence:

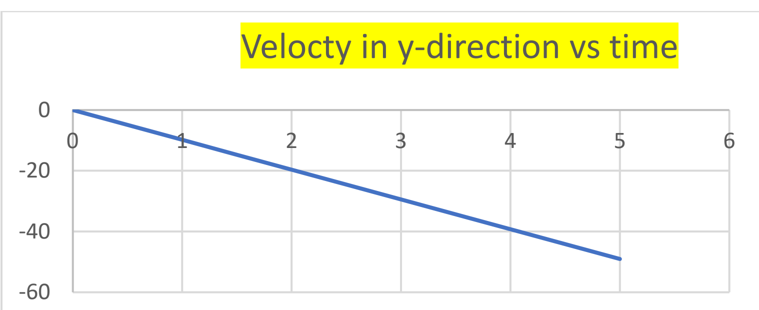
The initial positionfor both rocks is/are the same, so the vertical displacement to fall is also the same. Since both rocks are in the air for the same amount of time, the rock that is going horizontally faster, Rock C goes further.

Note: Let's assume that $v_x = 5m/s(\hat{i})$ and that it hits the ground after 5 seconds

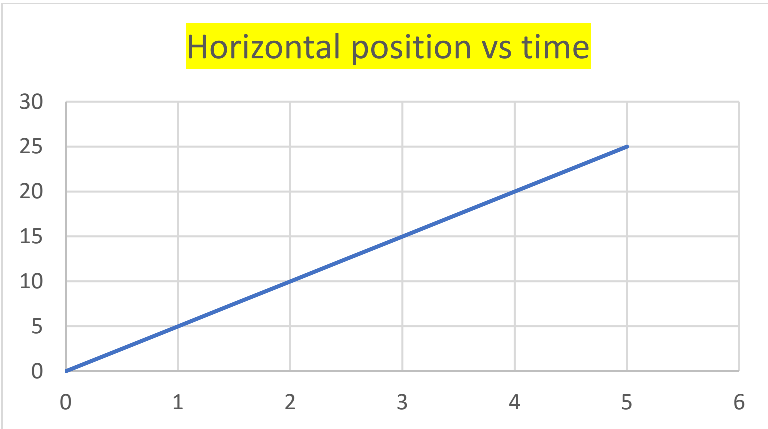


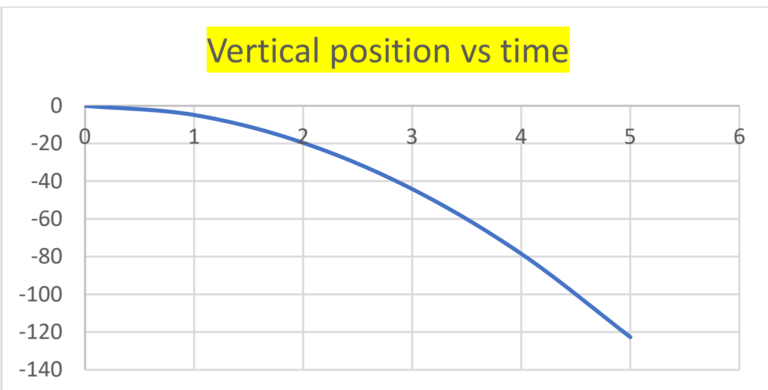




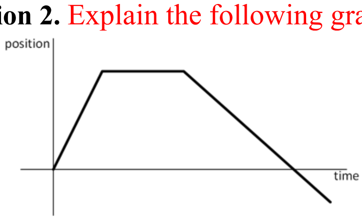


The slope of this line is the acceleration of the object





Question 2. Explain the following graphs (position-time; velocity-time, acceleration-time):



Graph 1: There are three parts to this Position vs Time graph.

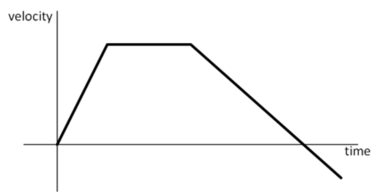
1: Object moves with constant positive velocity

2: Object remains at rest
3: Objects moves with constant negative velocity

We can describe it as a person walking in a straight line. The person moves to the right (positive constant velocity) for a given period of time and then stops. The person remains at rest for a given period of time and then walks to the left (negative constant velocity).

The slope of each line represents the velocities, which means the person walked with a greater speed in the positive direction.

The person ended up to the left of the starting point, the displacement for the entire period of time is negative.



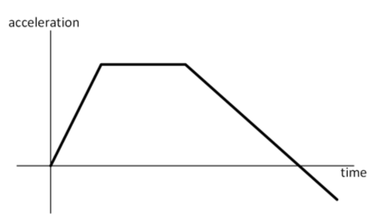
Graph 2: There are three parts to this Velocity vs Time graph.

1: Object moves in positive direction with a constant positive acceleration.
2: Object moves with constant positive velocity
3: Objects moves with constant negative acceleration.

We can describe this as you on a flat calm lake kayaking. You start from rest and move to the right, speeding up with a constant acceleration. You reach a certain velocity and stay at that velocity for a given amount of time. You then begin to paddle in the opposite direction causing you to slow down with a constant negative acceleration. You eventually come to a stop and then for short period of time, kayak in the negative direction.

The slope of each line represents the acceleration. We can say that the you "sped up" at a greater rate than you "slowed down"

The area between the time axis and the lines represent displacement, your displacement was positive. Most of the time you were moving to the right, only at the very end did you kayak to the left for a short period of time.



Graph 3: There are three parts to this Acceleration vs Time graph.

1: Object starts with zero acceleration and then has a positive acceleration, increasing at a constant rate.
2: Object moves with a positive constant acceleration.
3: Object moves with a positive acceleration, decreasing at a constant rate. Object momentarily has a zero acceleration and then has a negative acceleration.

This graph is difficult to explain as we do not know the direction of the object. It could be moving in either direction with any given speed at t = 0 seconds.

Let's assume a car is initially moving in the positive direction with a constant velocity at t = 0 seconds. The car begins to accelerate with an increasing acceleration. The car then accelerates with a constant value. The car continues to accelerate, but with a decreasing value. Eventually the car stops accelerating for a moment and then begins to accelerate in the negative direction.

Keeping with our assumption at t = o seconds, we can say the car moved in the positive direction for the whole time period, its velocity was always positive.

Question 3.
Two identical balls are at rest and side by side at the top of a hill. You let one ball, A, start rolling down the hill. A little later you start the second ball, B, down the hill by giving it a shove. The second ball rolls down the hill along a line parallel to the path of the first ball and passes it. At the instant ball B passes ball A:

a) it has the same position and the same velocity as A.
b) it has the same position and the same acceleration as A.
c) it has the same velocity and the same acceleration as A.
d) it has the same displacement and the same velocity as A.
e) it has the same position, displacement and velocity as A.

Both balls are on the same slope so they have the same acceleration. When the balls are at the same position they have moved the same displacement. Ball B must be moving faster as it "caught up" to ball A, so their velocities differ.

Answer: B

Question 4. Which of the following situations is impossible at any time?
a) An object has velocity directed east and acceleration directed west.
b) An object has velocity directed east and acceleration directed east.
c) An object has zero velocity but nonzero acceleration.
d) An object has constant nonzero acceleration and changing velocity.
e) An object has constant nonzero velocity and changing acceleration.

For A and B your direction of velocity and acceleration are irrelevant. A car can be going in any direction and be accelerating in any direction. For C, imagine a ball throw up. It comes to rest for a moment (velocity is zero) but is still accelerating down. D is fine. E claims to have an object accelerating with a constant velocity, a contradiction.

Answer: E

Question 5. Use this situation to answer the next three questions (i-ii-iii):
Consider a coin which is tossed straight up into the air. After it is released, it moves upward, reaches its highest point and falls back down again.

i) While the coin is moving upward after being released, what would be its acceleration? Take up to be the positive direction.

a) The acceleration is in the positive direction and increasing.
b) The acceleration is in the positive direction and decreasing.
c) The acceleration is in the negative direction and constant.
d) The acceleration is in the positive direction and constant.
e) The acceleration is in the negative direction and decreasing.
f) The acceleration is in the negative direction and increasing.
g) The acceleration is zero.

The acceleration of the coin is negative for the whole time the coin is in the air. Yes, it has a negative acceleration even when it comes to a stop.

Answer: C

ii) What would be its acceleration while the coin is at its highest point?
a) The acceleration is in the positive direction and increasing.
b) The acceleration is in the positive direction and decreasing.
c) The acceleration is in the negative direction and constant.
d) The acceleration is in the positive direction and constant.
e) The acceleration is in the negative direction and decreasing.
f) The acceleration is in the negative direction and increasing.
g) The acceleration is zero.

The acceleration of the coin is negative for the whole time the coin is in the air. Yes, it has a negative acceleration even when it comes to a stop.

Answer: C

iii) What would be its acceleration while the coin is moving downward?
a) The acceleration is in the positive direction and increasing.
b) The acceleration is in the positive direction and decreasing.
c) The acceleration is in the negative direction and constant.
d) The acceleration is in the positive direction and constant.
e) The acceleration is in the negative direction and decreasing.
f) The acceleration is in the negative direction and increasing.
g) The acceleration is zero.

The acceleration of the coin is negative for the whole time the coin is in the air. Yes, it has a negative acceleration even when it comes to a stop.

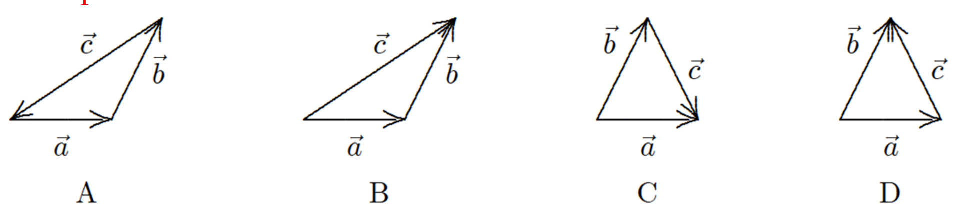
Answer: C

Question 6. We say the displacement of an object is a vector quantity. Our best justification for this assertion is:

a) Displacement can be specified by a magnitude and a direction.
b) Operating with displacements according to the rules for manipulating vectors leads to results in agreement with experiments.
c) A displacement is obviously not a scalar.
d) Displacement can be specified by three numbers.
e) Displacement is associated with motion.

Answer: A

Question 7. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{c} = \vec{b} - \vec{a}$. Which diagram below illustrates this relationship?



E. None of these

We can manipulate the formula to match the diagram, remember that we add vectors tip to tail:

$$\vec{c} = \vec{b} - \vec{a}$$
$$\vec{b} = \vec{a} + \vec{c}$$

We add vectors tip to tail. Answer: D

Question 8. Two cars racing on the Gilles Villeneuve circuit start side by side and begin the race at the same time. As soon as the winning car returns to the start/finish line, both cars stop. The losing car stops half a lap behind the winning car.

(a) Which car had the larger average speed? Explain your answer in one sentence

The winning car has a greater "distance travelled" and will therefore have a greater average speed.

(b) Which car had the larger average velocity? Explain your answer in one sentence.

The winning car with have an average velocity of zero as it has a displacement of zero, while the losing car will have a non-zero average velocity. Losing care will have larger average velocity.

PROBLEMS

1. A ball is dropped from a height of 3 m and rebounds from the floor to a height of 2 m.

a) What is the velocity of the ball just as it reaches the floor?

Before we start any calculations let’s agree that the velocity must be negative.

The acceleration of the ball is constant so we can use our linear motion equations. Be careful when you take the square root of a value, in this case the final velocity will be in negative y - direction

$$v_{fy}^{2} = v_{iy}^{2} + 2a_{y}(\Delta y)$$
$$v_{fy}^{2} = 0 + 2(-9.81)(-3)$$
$$v_{fy}^{2} = 58.86$$
$$v_{fy} = 7.67(m/s)(-\hat{j})$$

The velocity of the ball just as it reaches the floor is $v_{fy} = 7.67(m/s)(-\hat{j})$

b) What is the velocity just as it leaves the floor?

Again, the acceleration is constant. We expect a positive value of the initial velocity of the ball as it leaves the floor. We know the displacement, the acceleration and the final velocity.

$$v_{fy}^{2} = v_{iy}^{2} + 2a_{y}(\Delta y)$$
$$0 = v_{iy}^{2} + 2(-9.81)(2)$$
$$v_{fy}^{2} = 39.24$$
$$v_{fy} = 6.26(m/s)(\hat{j})$$

The velocity of the ball just as it reaches the floor is $v_{fy} = 6.26(m/s)(\hat{j})$

c) If it is in contact with the floor for 0.02s, what are the magnitude and direction of its average acceleration during this interval?

Note the term “average acceleration” is being used. As the ball compresses, the force of the floor acting on the ball gets larger. Via Newton and $\vec{F} = m\vec{a}$, if the force is changing then so is the acceleration.

Force is in positive y-direction so we expect value to be positive. Be aware of the double negatives.

$$a_{y(ave)} = \frac{v_{fy} - v_{iy}}{\Delta t}$$

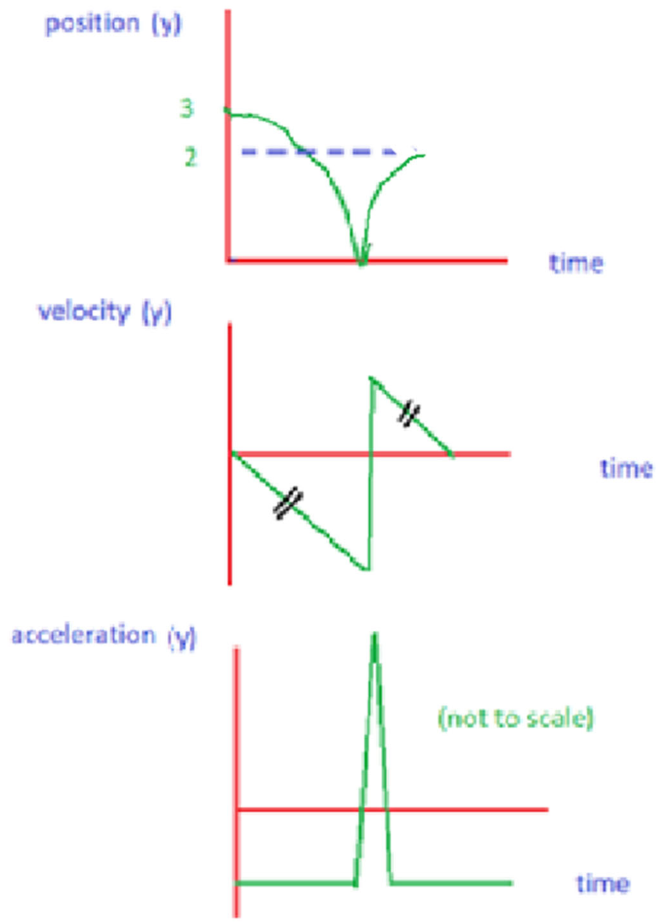
$$a_{y(ave)} = \frac{6.26 - (-7.67)}{.02}$$

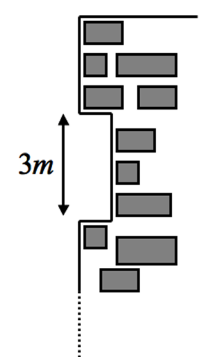
$$a_{y(ave)} = 696.5(m/s^2)(\hat{j})$$

d) Sketch position-time, velocity-time and acceleration-time graphs for the motion of the ball during the drop, the hitting of the floor and the upward motion

Be aware that these are rough sketches and not to scale. The 0.02 seconds is quite difficult to show on any of the graphs. What we are really interested in is the idea of the sketches, the understanding of what is happening.

For the P vs t graph, the acceleration of the object is -9.81 m/s^2 while going down and then up (curved lines), but not while in contact with the floor. On the V vs T graph, you will see this with the parallel lines.





2. A ball dropped from the roof of a building takes 0.25 s (from when it reaches the top edge of the window) to pass the window, which is 3m long.

a) How fast is it moving as it passes the top edge of the window?

The wording of this question is important. Previous students did not read it properly and wrongly assumed the initial velocity of the ball to be zero when is encounters the top of the window edge.

The velocity of the ball is negative as the ball is moving down. The displacement is also negative as is the acceleration.

$$v_{fy}^{2} = v_{iy}^{2} + 2a_y(\Delta y)$$

I was going to use this formula until I realized that I have two unknowns. Time is given so let's move to next possible formula.

$$\Delta y = v_{iy}(t) + \frac{1}{2}a_y t^2$$
$$-3 = v_{iy}(.25) + \frac{1}{2}(-9.81)(.25)^2$$
$$v_{iy} = 10.77(m/s)(-\hat{j})$$

The velocity of the ball as it passes the top edge of the window is $v_y = 10.77(m/s)(-\hat{j})$

b) How far below the roof is the top edge of the window?

The object started at rest and after raveling a given distance had a velocity of $v_y = 10.77(m/s)(-\hat{j})$.
We know the acceleration and can solve for the displacement:

$$v_{fy}{}^2 = v_{iy}{}^2 + 2a_y(\Delta y)$$
$$(10.77)^2 = 0 + 2(-9.81)(\Delta y)$$
$$\Delta y = 5.91m(-\hat{j})$$

The top of the window is 5.391m below the roof.

3. A toad undergoes the following displacements: $\vec{A} = 3.0m@110^o$, $\vec{B} = 5.0m@200^o$, $\vec{C} = 6.0m@295^o$

a) What is the total displacement of the toad? Express your answer in unit vector notation.

Vectors, when added mathematically, must be in component form.

$\vec{A} = 3.0m@110^o$ $\vec{B} = 5.0m@200^o$ $\vec{C} = 6.0m@295^o$

$\vec{A} = -1.03(\hat{i}) + 2.82(\hat{j})$ $\vec{B} = -4.70(\hat{i}) - 1.71(\hat{j})$ $\vec{C} = 2.54(\hat{i}) - 5.44(\hat{j})$

$$\vec{d} = total\_displacement$$
$$\vec{d} = \vec{A} + \vec{B} + \vec{C}$$
$$\vec{d} = (-1.03(\hat{i}) + 2.82(\hat{j})) + (-4.70(\hat{i}) - 1.71(\hat{j})) + (2.54(\hat{i}) - 5.44(\hat{j}))\,\mathrm{m}$$
$$\vec{d} = (-3.19(\hat{i}) - 4.33(\hat{j}))m$$

The total displacement of the toad is $\vec{d} = (-3.19(\hat{i}) - 4.33(\hat{j}))m$

b) The three displacements happened over a 20.0-s time interval. Express the toad's average velocity in polar notation.

We will work in component form and then convert to polar form at the end.

$$\vec{v}_{ave} = \frac{displacment}{time} = \frac{(-3.19(\hat{i}) - 4.33(\hat{j}))m}{20s}$$
$$\vec{v}_{ave} = (-0.160\hat{i} - 0.217\hat{j})m/s$$

$$\vec{v}_{ave} = 0.27m/s @ 233.60^{o}$$

The toad's average velocity during the 20 second interval is $\vec{v}_{ave} = 0.27m/s @ 233.60^{o}$

c) What is the toad's average speed over the 20.0-s time interval?

Average speed is very different than average velocity. Speed refers to a scale value so instead of displacement we are looking at the total distance travelled by the toad. "Distance travelled" will always be positive, regardless of the direction the toad is moving.

The total distance travelled is $d = 3 + 5 + 6 = 14(m)$

$$speed_{ave} = v_{ave} = \frac{toal\_dis\tan ce\_travelled}{time}$$
$$speed_{ave} = v_{ave} = \frac{14}{20} = 0.7(m/s)$$

The toad's average speed during the 20 second interval is $speed_{ave} = v_{ave} = 0.7(m/s)$

4. A ball is moving with a velocity $\vec{v}_i = (3\hat{i} - 4\hat{j})(m/s)$. Ten seconds later it has a velocity $\vec{v}_f = (5\hat{i} + 2\hat{j})(m/s)$. If the acceleration is constant,

a) What is the acceleration of the object? Sketch a vector diagram.

We use the linear motion formula $\vec{v}_f = \vec{v}_i + \vec{a}(t)$ to solve

$$\vec{v}_f = \vec{v}_i + \vec{a}(t)$$
$$(5\hat{i} + 2\hat{j}) = (3\hat{i} - 4\hat{j}) + \vec{a}(10)$$
$$\vec{a} = \frac{2\hat{i} + 6\hat{j}}{10}$$
$$\vec{a} = (0.2\hat{i} + 0.6\hat{j})\,m/s^2$$

The acceleration is $\vec{a} = (0.2\hat{i} + 0.6\hat{j})\,\mathrm{m/s^2}$

b) What is the displacement of the object during the ten second interval?

We use the linear motion formula $\Delta\vec{d} = \vec{v}_i(t) + \frac{1}{2}\vec{a}(t^2)$ to solve.

We can solve it in the x-direction or y-direction independently or we can solve for both at once.

$$\Delta\vec{d} = \vec{v}_i(t) + \frac{1}{2}\vec{a}(t^2)$$
$$\Delta\vec{d} = (3\hat{\imath} - 4\hat{\jmath})(10) + \frac{1}{2}(0.2\hat{\imath} + 0.6\hat{\jmath})(10^2)$$
$$\Delta\vec{d} = (30\hat{\imath} - 40\hat{\jmath}) + \frac{1}{2}(20\hat{\imath} + 60\hat{\jmath})$$
$$\Delta\vec{d} = (40\hat{\imath} - 10\hat{\jmath})m$$

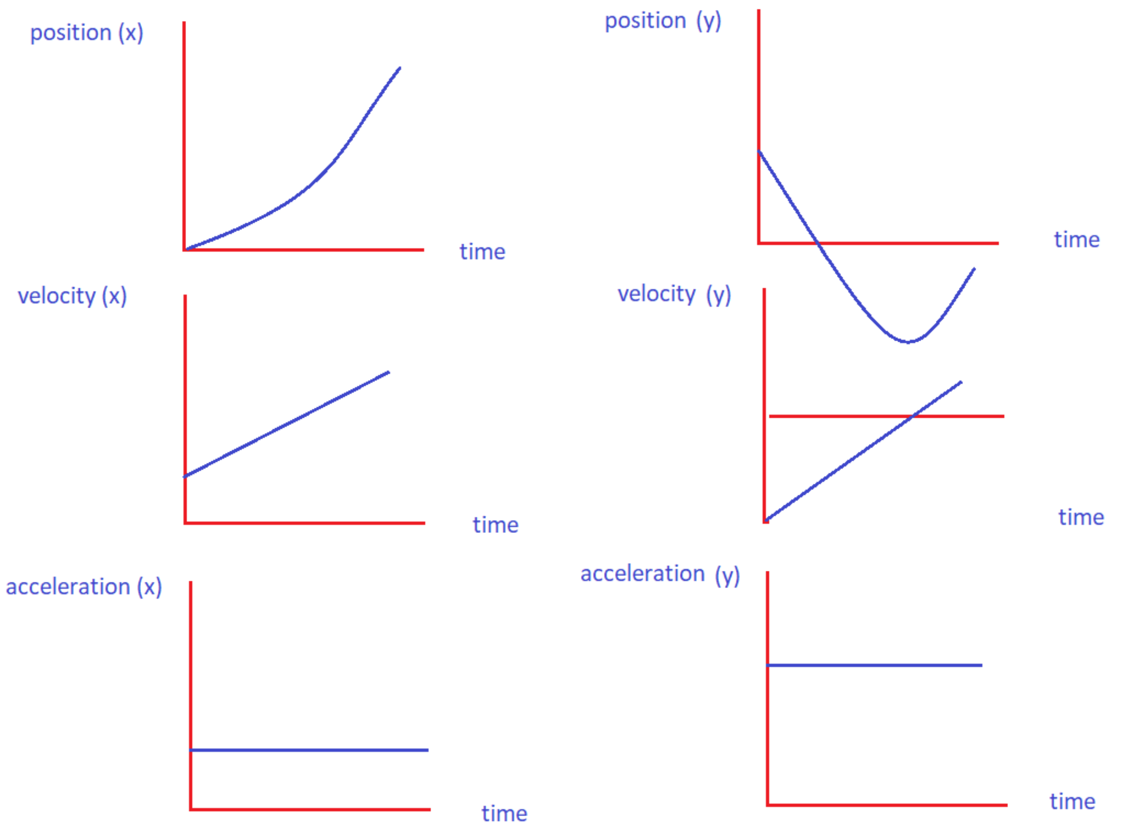
If you wanted to, you could solve for the displacement in each direction independently. It takes more time and more math, but it does work. Note how I use subscripts to denote the direction:

$x - direction$	$y - direction$
$\Delta x = \vec{v}_{ix}(t) + \frac{1}{2}\vec{a}_x(t^2)$	$\Delta y = \vec{v}_{iy}(t) + \frac{1}{2}\vec{a}_y(t^2)$
$\Delta x = (3\hat{\imath})(10) + \frac{1}{2}(0.2\hat{\imath})(10^2)$	$\Delta y = (-4\hat{\jmath})(10) + \frac{1}{2}(0.6\hat{\jmath})(10^2)$
$\Delta x = (30\hat{\imath}) + \frac{1}{2}(20\hat{\imath})$	$\Delta y = (-40\hat{\jmath}) + \frac{1}{2}(60\hat{\jmath})$
$\Delta x = (40\hat{\imath})m$	$\Delta y = (-10\hat{\jmath})m$

We get the same answer, use which ever one you are comfortable with.

c) Sketch the position-time, velocity-time and acceleration-time graphs for the x- and the y-direction (6 sketches total)

Note that we were not given an initial positions, so we can randomly choose where we want to start in x and y directions



5. An object is set in motion with an initial velocity $\vec{v}_i = 100m/s @ 53^o$ If the acceleration of the object is given by $\vec{a} = -10m/s^2(\hat{j})$, find:

a) The position of the object after 12 seconds if the initial position is the origin.

We start by converting the polar form vector to component form. Before we jump into the math, we recognize that an angle of 53 degrees will have a vertical component bigger than the horizontal.

$$\vec{v}_i = 100m/s @ 53^o$$

$$\vec{v}_i = (60.18\hat{i} + 79.86\hat{j})m/s$$

I will solve in both direction at once using $\Delta\vec{d} = \vec{v}_i(t) + \frac{1}{2}\vec{a}(t^2)$

$$\Delta\vec{d} = \vec{v}_i(t) + \frac{1}{2}\vec{a}(t^2)$$
$$\Delta\vec{d} = (60.18\hat{i} + 79.86\hat{j})(12) + \frac{1}{2}(-10\hat{j})(12^2)$$
$$\Delta\vec{d} = (722.16\hat{i} + 958.32\hat{j}) - 720\hat{j}$$
$$\Delta\vec{d} = (722.16\hat{i} + 238.32\hat{j})m$$

We just solved for the displacement. If we assume the object started at the origin, the position of the object will be $(722.16\hat{i} + 238.32\hat{j})m$

b) The velocity of the object at this time.

I can use the formula $\vec{v}_f = \vec{v}_i + \vec{a}(t)$ to solve. I will solve in both direction at once in each direction independently.

$$\vec{v}_f = \vec{v}_i + \vec{a}(t)$$
$$\vec{v}_f = (60.18\hat{i} + 79.86\hat{j}) + (-10\,\hat{j})(12)$$
$$\vec{v}_f = (60.18\hat{i} - 40.14\hat{j})m/s$$

$x - direction$

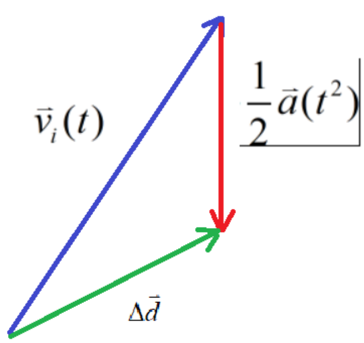
$$\vec{v}_{fx} = \vec{v}_{ix} + \vec{a}_x(t)$$
$$\vec{v}_{fx} = 60.18 + (0)(12)$$
$$\vec{v}_{fx} = 60.18m/s(\hat{i})$$

$y - direction$

$$\vec{v}_{fy} = \vec{v}_{iy} + \vec{a}_y(t)$$
$$\vec{v}_{fy} = 79.86 + (-10)(12)$$
$$\vec{v}_{fy} = -40.14$$

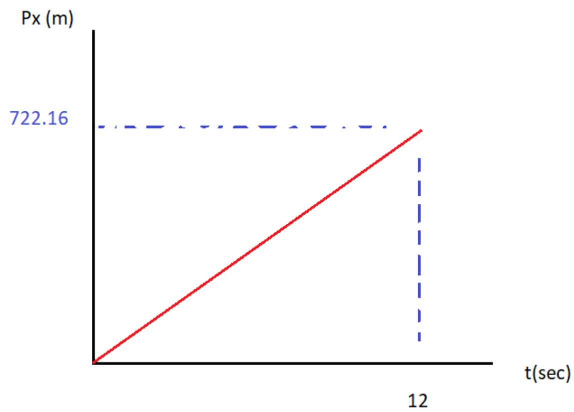
Both methods give the same answer, find what you are comfortable with.

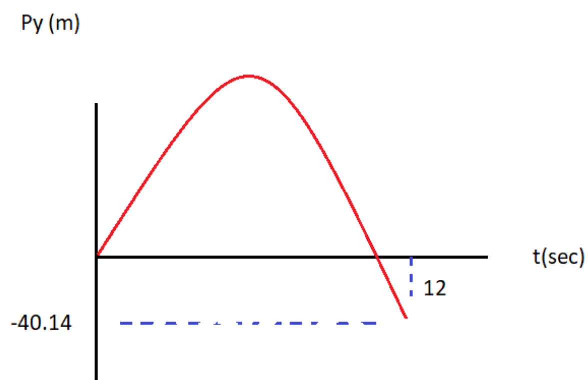
c) Sketch a vector diagram to demonstrate part (a)

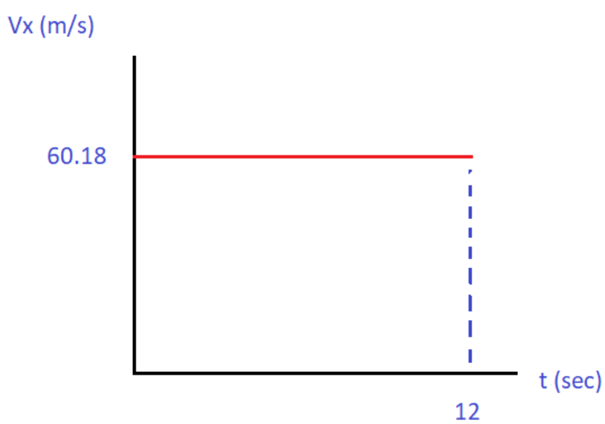


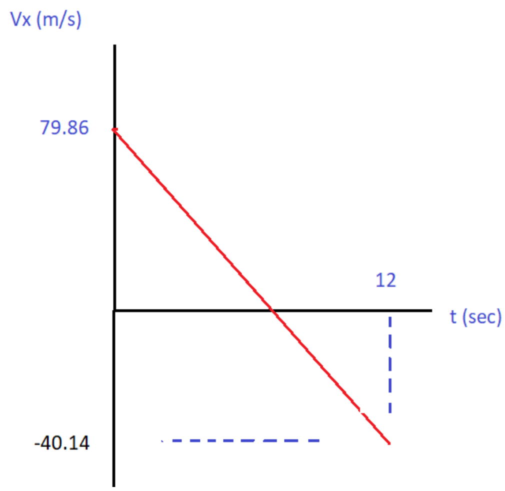
d) Sketch the position-time, velocity time and acceleration-time graphs in x and y (6 graphs total)

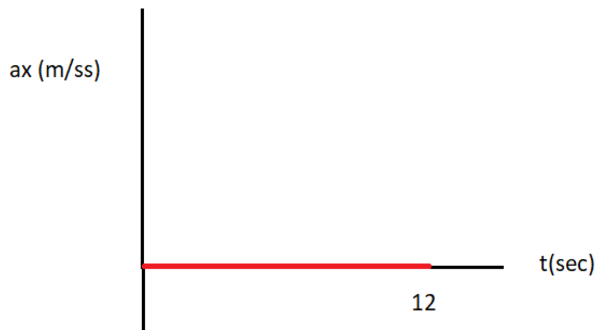
Note that there is no acceleration in the horizontal direction.

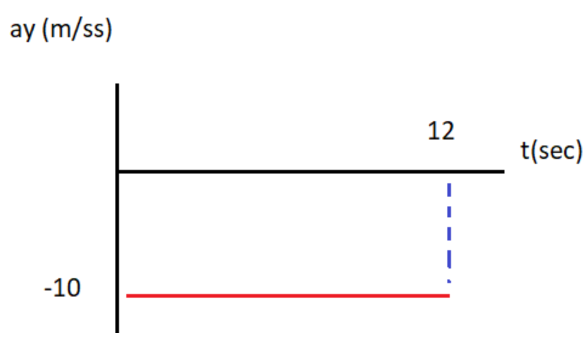












6. In a game of baseball on Mars, a batter hits a ball at a height of 1.50 m above the ground so that its angle of projection is 45.0° and its initial speed is 15.0 m/s. The ball is hit far down the left field line where an 8.00 m high fence is located 100.0 m from home plate. The acceleration due to gravity on Mars is 3.70 m/s^2. Will the ball clear the fence?

Solution

This is a classic projectile motion question. Be aware that the ball starts at a height of 1.5 meters, this is where past students trip up. The horizontal acceleration is zero and the vertical acceleration is

$$a_y = -3.70\ \frac{m}{s^2}$$

There are two ways to solve this problem, I will solve them both for you. Regardless of which solution we use, the velocity vector must be in component form:

$$\vec{v}_i = 15\frac{m}{s}\ at\ 45°$$
$$\vec{v}_i = (10.6\hat{\imath} + 10.6\hat{\jmath})\frac{m}{s}$$

Yes, the 45-degree angle gives the same vertical and horizontal values.

Solutions 1: Let's solve for the time it takes the ball to reach 100 meters. Be aware that it may never reach 100 meters, the math will tell us. With that time, we will find the height of the ball. If it is above the 8 m mark it will clear the fence.

$$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$$
$$\Delta x = v_{ix}(t) + 0$$
$$100 = 10.6(t)$$
$$t = 9.43sec$$

The ball will reach the 100m horizontally at 9.43 seconds. Now we use this result to find the displacement in the vertical direction. This is where the acceleration due to gravity on Mars comes in.

$$\Delta y = v_{iy}(t) + \frac{1}{2}a_y t^2$$
$$\Delta y = 10.6(9.43) + \frac{1}{2}(-3.70)(9.43)^2$$
$$\Delta y = 99.96 - 164.5$$
$$\Delta y = -64.5m(\hat{\jmath})$$

The ball had a vertical displacement of -64.5 meters, If we count the 1.5m start position this would mean that the ball's final position is 63.5m below ground. This silly result tells us that the ball does not make it over the fence.

Solution 2: We will find the time it takes for the ball to reach the height of 8 m above the ground. As it starts at 1.5 m, we want to find out the time it takes for a vertical displacement of 6.5 meters. We expect two values of "t", one on the way up and another on the way down.

$$\Delta y = v_{iy}(t) + \frac{1}{2}a_y t^2$$
$$6.5 = 10.6(t) + \frac{1}{2}(-3.70)(t)^2$$
$$quadratic\_equation$$
$$-1.85t^2 + 10.6t - 6.5 = 0$$
$$t_1 = 0.703\ s\ and\ t_2 = 5.02s$$

We are interested in the ball on the way down at 5.02 seconds. Now let's see where the ball is horizontally after 5.02 seconds.

$$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$$
$$\Delta x = 10.6(5.02) + 0$$
$$\Delta x = 53.2m(\hat{\imath})$$

This tells us that when the ball was 8m above the ground on its way down, it had travelled 53.2 m horizontally. This is not far enough. The ball does not clear the fence.

7. Frustrated with rising prices, Robin Hood and his not-so-merry men attacked the Sheriff of Nottingham's castle with a catapult. Their first boulder is shot with a velocity of 40 m/s at an angle of 30° above the horizontal from a hill which is 35 m higher than the surrounding countryside. The top of the castle wall is 50 m above the surrounding countryside and the boulder just grazes the top of the wall on its way down.

a) Calculate the time of flight from the ground to the castle.

This is a "wordy" question, be careful and read more than once. The vertical displacement is 15m. Once again, let's get our initial velocity in component form:

$$\vec{v}_i = 40m/s@30^o$$
$$\vec{v}_i = (34.64\hat{i} + 20\hat{j})m/s$$

Yes, at an angle of 30 degrees we expect our horizontal value to be bigger. We can solve using the equation $\Delta y = v_{iy}(t) + \frac{1}{2}a_y t^2$.

$$\Delta y = v_{iy}(t) + \frac{1}{2}a_y t^2$$
$$15 = 20(t) + \frac{1}{2}(-9.81)t^2$$
$$0 = -4.905t^2 + 20t - 15$$
$$t = 0.99\_and\_t = 3.09$$

It will take 3.09 seconds for the boulder to grazed the top of the wall.

b) How far has the boulder traveled horizontally?

$$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$$
$$\Delta x = (34.64)(3.09) + 0$$
$$\Delta x = 107.04m(\hat{i})$$

The boulder has travelled $\Delta x = 107.04m$ horizontally.

c) What is the maximum height the boulder reaches above the surrounding countryside?

Maximum height occurs when the velocity in the vertical direction is zero. We will solve for the vertical displacement and then take into consideration the starting position.

$$y-direction$$
$$\vec{v}_{fy}{}^2 = \vec{v}_{iy}{}^2 + 2\vec{a}_y(\Delta y)$$
$$0 = (20)^2 + 2(-9.81)(\Delta y)$$
$$\Delta y = 20.39m(\hat{j})$$

The boulder started 35 m above the surrounding countryside, so the maximum height is 55.39 m.

d) What is the boulder's velocity just as it reaches the castle wall?

The horizontal velocity of the boulder will remain constant at $\vec{v}_x = (34.64\hat{i})m/s$. We need to solve the vertical velocity.

$$y-direction$$
$$\vec{v}_{fy}{}^2 = \vec{v}_{iy}{}^2 + 2\vec{a}_y(\Delta y)$$
$$\vec{v}_{fy}{}^2 = 20^2 + 2(-9.81)(15)$$
$$\vec{v}_{fy} = \pm 10.28\hat{j}(m/s)$$

The boulder is on the way down so $\vec{v}_{fy} = -10.28\hat{j}(m/s)$

The final velocity of the boulder is $\vec{v}_f = (34.64\hat{i} - 10.28\hat{j})\,\mathrm{m/s}$

8. A baseball just clears a 3 m wall that is 120 m from home plate.

a) If it leaves the bat at 45° and 1.2 m above the ground, what must be its initial velocity?

We are given both the horizontal and vertical displacements: $\Delta x = 120\hat{i}(m)$, $\Delta y = 1.8\hat{j}(m)$

As the acceleration is zero in horizontal direction I will start in x-direction.

$$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$$
$$120 = v_{ix}(t) + 0$$
$$120 = (v\cos 45)(t)$$

We have two unknowns and cannot go any further. Let's move onto y-direction and see where it takes us.

$$\Delta x = v_{ix}(t) + \frac{1}{2}a_x t^2$$
$$1.8 = (v\sin 45)(t) - 4.905t^2$$

We have the same two unknowns. We can now substitute and solve:

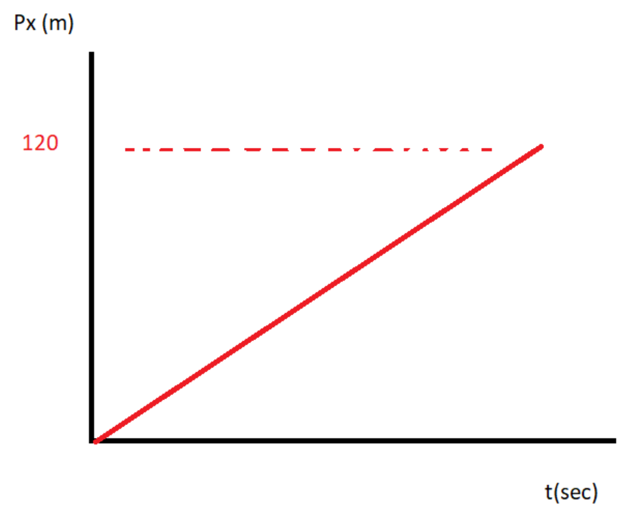
$$1.8 = (v\sin 45)(t) - 4.905t^2 \_ and \_ 120 = (v\cos 45)(t)$$
$$1.8 = (v\sin 45)(\frac{120}{v\cos 45}) - 4.905(\frac{120}{v\cos 45})^2$$
$$1.8 = \tan 45(120) - \frac{141264}{v^2}$$
$$1.8 = 120 - \frac{141264}{v^2}$$
$$\frac{141264}{v^2} = 120 - 1.8$$
$$v^2 = 1195.13$$
$$v = 34.57 m/s$$

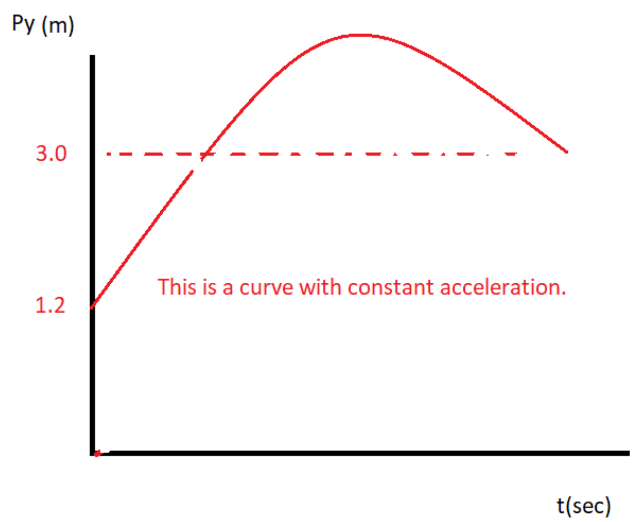
We just solved for the magnitude of the velocity. We can now write it as a vector:

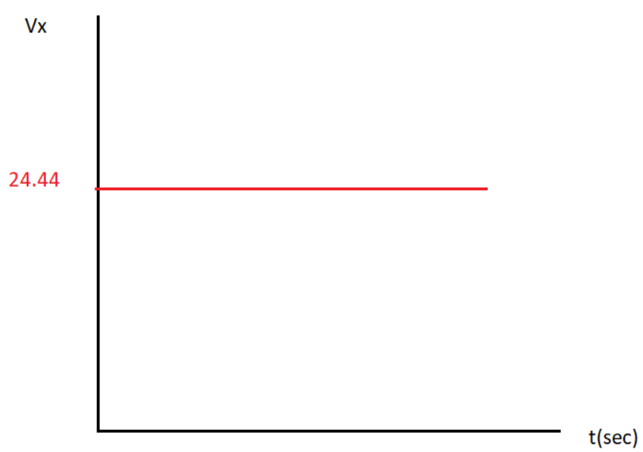
$$\vec{v}_i = 34.57 m/s @ 45^o$$

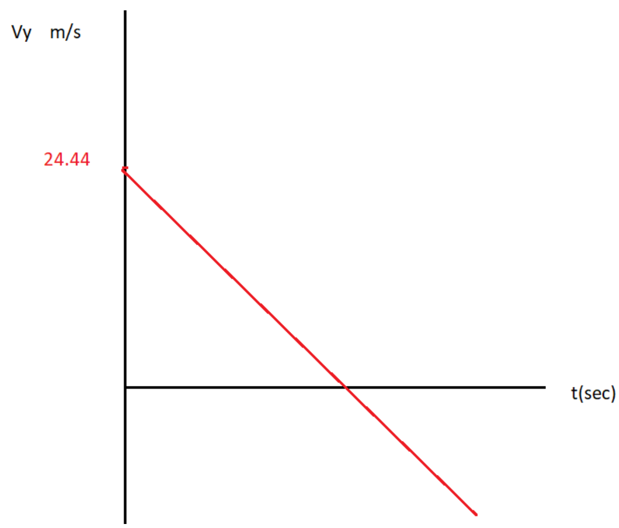
b) Sketch position-time and velocity time graphs for the baseball in x and y (4 sketches total)

These are simple sketches. I did not solve for time and therefore will not give the time of maximum height or time to reach the 3 m height.


Px (m)
120
t(sec)


Py (m)
3.0
1.2
This is a curve with constant acceleration.
t(sec)


Vx
24.44
t(sec)


Vy m/s
24.44
t(sec)