Physics NYC

Problem Set #7: Relativity

Solutions

A01: "C"

A02: "B" note that "moving clocks appear to run slowly"

A03: "C" This is a good one and worthy of being fully understood. The key to look for is the ***definition*** of proper time and proper length. In whose frame was the clock stationary? In whose frame was the 'block' stationary?

A04: "A"

A05: "A" The key question to ask is: Is there a situation in which the other F.O.R. would have to be moving faster than c in order for B to precede A? e.g. What if in your reference frame A was "throwing a rock" and B was "window breaking"? Try to draw F.O.R. in which B precedes A. What is the speed of this F.O.R.? Causality and the laws of physics prevent this from happening. A frame in which A precedes B definitely exists.

A06: "D" You should not need any calculations to eliminate the other options.
(A tip: never choose an answer where something is going faster than the speed of light!)

A07: "B"

A08: "B" Think about gamma.

A09: This is possible when you consider time dilation. In the frame of reference of the astronaut, the trip may only take six years, but to those observing on Earth, it would take longer. How can the two times both make sense? Observers on Earth see the spaceship travel 8 ly in time Δt. For the astronaut (who is stationary!) the 'stick' joining Earth to Sirius is what is moving, and length contraction makes it considerably shorter than eight ly. This shorter stick passes by the ship in $\Delta\tau = 6$ years.
[For bonus marks: how fast is the astronaut traveling?]

A10: The correct answer to this is "it depends". For an observer on the track, it is possible to completely enclose the train in a 90-m tunnel, since, according to the observer, the train is only 80-m long. However, for someone on the train, it would be impossible to enclose the train in that tunnel. To the train passengers, the tunnel would appear to be less than 90-m long, thus you cannot fit a 100-m train in it!

How can both answers be reconciled?

To the person on the track, both doors were closed at the same time (for a brief instant before the train burst through the second door). To the person on the train, the front of the train burst through the second door well before the first door closed behind the back of the train.

A11: Recall from your NYB days that the unit "eV" (electron-volt) is a measure of energy. Relativity shows that mass and energy are equivalent: $m = \frac{E}{c^2}$. 0.511 MeV/c^2 is identical to the mass of an electron in kg.

[To check: $m = \frac{0.511\times10^6 \times 1.6\times10^{-19}\,\text{J}}{\left(3\times10^8\text{m/s}\right)^2} = 9.08\times10^{-31}\text{kg}$]

B01: a) The lifetime for the pion given is the proper time Δτ= 26 ns. We need to find Δt. To make life easier, let's first find γ:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.99^2}} = 7.089$$

So, the next step is simply to find Δt:

$$\Delta t = \gamma\Delta\tau = 7.089\times 26 = 184.3\ ns$$

b) To the Earth observer, the pion travels for 184.3 ns and is traveling at 0.99c. Since both values are according to the same frame of reference, we can simply say:

$$v = \frac{\Delta x}{\Delta t} \quad \Rightarrow \quad \Delta x = v\Delta t = (0.99)(3\times10^8)(184.3\times10^{-9}) = 54.7\ m$$

c) To the pion, it travels for 26 ns and, again, is traveling at 0.99c, so we can again simply calculate:

$$v = \frac{\Delta x}{\Delta \tau} \quad \Rightarrow \quad \Delta x = v\Delta\tau = (0.99)(3\times10^8)(26\times10^{-9}) = 7.72\ m$$

d) This is simply a case of relativity in action, how different observers, in different frames of reference observe different times and lengths for similar events.

B02: Okay, we know the proper length of the meter stick, l_o = 1.0 m and the contracted length, l = 0.50 m. First step is to find gamma, and then solve for velocity.

$$l = \frac{l_0}{\gamma} \quad \Rightarrow \quad \gamma = \frac{l_0}{l} = \frac{1.0}{0.5} = 2$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \quad \Rightarrow \quad v = \frac{\sqrt{\gamma^2-1}}{\gamma}c = 0.866c$$

B03: Only the length in the direction of motion is contacted, thus the 80 m base of the triangle is unaffected. First, some geometry: what is the height of the triangle from 80-m base to tip?

$$a^2 + b^2 = c^2 \quad \Rightarrow \quad height = \sqrt{136^2 - (80/2)^2} = 130.0\,m$$

This 130-m length gets contracted:

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1 - v^2/c^2} = 130.0\sqrt{1 - 0.74^2} = 87.4\,m$$

NOTE: *This is a drastic over-simplification of the problem. In reality, the triangle would be skewed and twisted as it flew past. In addition, it would appear to us to be rotating, even though it is not. For the purposes of this course, you need not worry about the skewed results, but you should be aware that life at high speeds is not this easy!*

B04: The astronaut's pulse is 60 beats per minute, which is one per second. You can change the question to become: what does one second look like to an Earth observer? Let's start by solving for gamma:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.78^2}} = 1.598$$

Thus, one second would look like:

$$\Delta t = \gamma \Delta \tau = 1.598 \times 1 = 1.598\ s$$

Thus, each beat would be 1.598 seconds, or a pulse rate of a mere 37.5 beats/minute (very relaxed!).

B05:

Time measured from Earth $\Delta t = \frac{l_0}{v} = \frac{26.1\ c \cdot \text{yrs}}{0.95c}$
$= 27.5\ \text{yrs}$

Cathy will be 22 + 27.5 = 49.5 yrs old.

Time measured on space ship $\Delta \tau = \frac{\Delta t}{\gamma} = 27.5\sqrt{1 - 0.95^2}$
$= 8.59\ \text{yrs}$

John will be 22 + 8.6 = 30.6 yrs old.

[OR:

$$l = \frac{l_0}{\gamma} = 26.1\left(\sqrt{1-0.95^2}\right)$$
$$= 8.15 \text{ light yrs}$$

$$\Delta\tau = \frac{l}{v} = \frac{8.15\ c\cdot\text{yrs}}{0.95c}$$
$$= 8.58 \text{ yrs}$$

B06: Okay, before you begin any of these relativistic velocity questions, you need to get a feel for the answer. Since the original spacecraft is moving at 0.80c relative to the stationary umpire, then the "small round sphere" fired forward should appear to the umpire as ***traveling faster than 0.80c***. Further, the sphere cannot exceed the speed of light. Thus our answer must be between 0.80c and 1.0c.

Using the velocity transform equations:

$$u = \frac{u'+v}{1+vu'/c^2} = \frac{0.8c+0.7c}{1+0.7\times 0.8} = 0.962\,c$$

***<u>TIP</u>**: If you get confused about which is v and u' and such, take a look at the equation. It doesn't matter which one is which!*

B07: Power is "energy per unit time". Thus, in one second, the sun uses up 3.8×10^{26} J of energy. According to Einstein (smart guy):

$$E = mc^2 \quad \Rightarrow \quad m = E/c^2 = 3.8\times10^{26}/(3\times10^8)^2 = 4.22\times10^9\ kg$$

So the sun loses 4.22×10^9 kg/sec. Think about it this way: in about 100 seconds, the sun uses up the equivalent of the mass of the entire population of the Earth! That's a lot of energy!

B08: The two particles are approaching one another, so we use the same logic as per question B04; our answer must be between 0.85c and 1.0c.

$$u = \frac{v+u'}{1+vu'/c^2} = \frac{-0.55c-0.85c}{1+(-0.55)\times(-0.85)} = -0.954\,c$$

Since we are asked the velocity: $\vec{v} = -0.954c\,\vec{i}$

B09: We are given the kinetic energy of the proton.

(Let's assume that this is a relativistic energy level, and we cannot use ½mv². Try it and see if the velocity that it gives is greater than the speed of light).

The relativistic kinetic energy is given by:

$$K_{rel} = (\gamma - 1)mc^2 \quad \Rightarrow \quad \gamma = \frac{K_{rel}}{mc^2} + 1 = \frac{50\times10^9}{\frac{939\times10^6}{c^2}\times c^2} + 1 = 54.2$$

Using gamma, we can solve for velocity:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad v = \frac{\sqrt{\gamma^2 - 1}}{\gamma}c = 0.9998c \quad \text{(that's fast!)}$$

B10: a) The energy required is the kinetic energy, so we will be solving for that. First, let's find gamma:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.60^2}} = 1.25$$

Now find K:

$$K_{rel} = (\gamma - 1)mc^2 = (1.25 - 1)\times10^6 \times (3\times10^8)^2 = 2.25\times10^{22}\ J$$

b) Converting mass into energy:

$$E = mc^2 \quad \Rightarrow \quad m = E/c^2 = 2.25\times10^{22}/(3\times10^8)^2 = 250000\,kg$$