PROBLEM SET 6

QUANTUM PHYSICS

Part A

1. C
2. B $\quad p = \frac{h}{\lambda} \quad$ 2 x energy $\Rightarrow \frac{1}{2}\lambda \quad \left[E = \frac{hc}{\lambda}\right] \quad \therefore 2 \times p$
3. D x-rays travelling straight through (or reflected) are unchanged; the scattered x-rays (λ_3) have given some of their energy to electrons with which they collided.
4. C
5. B
6. C
7. D
8. E $\quad E_1 = \frac{n^2h^2}{8mL^2} \qquad E_2 = \frac{n^2h^2}{8(2m)(2L)^2} = \frac{1}{8}E_1$
9. A
10. C Longest $\lambda \Rightarrow$ smallest energy
    $E_{photon} = -\Delta E_{atom}$ Smallest ΔE_{atom}: Look for smallest 'jump' (but must finish on level n = 2 for Balmer series).

11. a) Vibrating electrons, atoms, molecules etc. can only have discrete values of energy

$$E_n = nhf$$

    where f is the natural frequency of oscillation.

    b) These oscillators can only emit or absorb energy in discrete packets.

12. An electron accelerating around a nucleus should continuously emit EM radiation (and with it, energy). As it loses energy it should spiral into the nucleus (just like a satellite losing energy due to atmospheric friction would spiral inward and eventually fall to the ground). As it falls into lower orbits it will revolve with greater and greater angular frequencies and therefore the radiation emitted will also have greater and greater frequencies. Result: As it spirals inward to the nucleus, a whole spectrum of EM frequencies will be emitted. (Another result: the lifetime of the orbit, and therefore of the atom, would only be a tiny fraction of a second. Hmmm…)

13. If an object is comparable in size (or smaller) to the wavelength of the waves that are hitting it, it will not be detected. The wavelengths of electrons at the energies used in the electron microscope are **orders of magnitude smaller** than the wavelengths of visible light and can therefore discern smaller details.

Part B

1. a) The kinetic energy of the released electrons is related to the energy of the illuminating photon and the work function (ϕ) :

$$K = \frac{hc}{\lambda} - \phi = \frac{(4.14\times10^{-15})(3\times10^{8})}{3.5\times10^{-7}} - 2.24 = 1.31eV$$

(Note that ϕ does not correspond to the phase constant!!)

b) When wavelengths are longer than the cutoff wavelength, the wavelengths will not have enough energy to extract any electrons. So K = 0.

$$\frac{hc}{\lambda} = \phi \Rightarrow \lambda = \frac{hc}{\phi} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2.24} = 5.54\times10^{-7}\,m$$

2. a) Planck's constant for this experiment can be found from the information given in the graph

$$K = hf - \phi$$

$$\frac{K}{e} = \frac{hf}{e} - \frac{\phi}{e} \qquad \left(\frac{K}{e} = V_o\right)$$

$$V_o = \left(\frac{h}{e}\right)(f) - \frac{\phi}{e} \quad \Rightarrow \quad \text{linear graph of } V_0 \text{ vs } f$$

where the slope corresponds to $\frac{h}{e}$

From graph: $slope = \dfrac{4.0\text{ V}}{1.1\times10^{15}\text{ Hz}} = 3.6\times10^{-15}\text{ V}\cdot\text{s}$

(Remember: Volts are Joules/Coulomb (J/C) or electron volts/elementary charge (eV/e))

$$h = (slope)(e) = 3.6\times10^{-15}\text{ eV}\cdot\text{s}$$

b) At $V_0 = 0$, $hf = \phi$

$\phi = \left(3.6\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(1.05\times10^{15}\text{ Hz}\right)$ (from graph)

= 3.8 eV

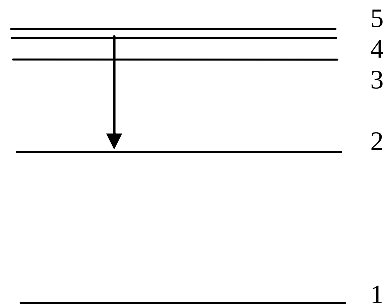
3. a) $E_{initial} = \frac{-13.6}{n_i^2} = -0.85 \Rightarrow n_i = 4$ and $E_{final} = \frac{-13.6}{n_f^2} = -3.4 \Rightarrow n_f = 2$

b) $E_{photon} = -\Delta E_{atom} = -(-3.4 - (-0.85)) = 2.55eV$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2.55} = 4.87\times10^{-7}m$$

c) Balmer

d)



4. In the Balmer series for hydrogen the final state $n_f = 2$

$$E_{photon} = -\Delta E_{atom}$$

from n = 3 to n = 2:

$$E_{photon} = \frac{13.6}{2^2} - \frac{13.6}{3^2} = 1.89eV$$

$$\lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15})(3\times10^{8})}{1.89} = 6.57\times10^{-7}m$$

from n = 4 to n = 2:

$$E_{photon} = \frac{13.6}{2^2} - \frac{13.6}{4^2} = 2.55eV$$

$$\lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2.55} = 4.87\times10^{-7}m$$

from n = 5 to n = 2:

$$E_{photon} = \frac{13.6}{2^2} - \frac{13.6}{5^2} = 2.86eV$$

$$\lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2.86} = 4.35\times10^{-7}m$$

b) (i) $\Delta E_{atom} = \frac{-13.6}{5^2} - \frac{-13.6}{4^2} = 0.306eV$ a photon with this energy can cause this transition

(ii) photon energy: $\frac{-13.6}{6^2} - \frac{-13.6}{5^2} = 0.166eV$

(iii) $\lambda = \frac{hc}{E}$ for $\lambda_{4\to5} = \frac{(4.14\times10^{-15})(3\times10^{8})}{0.306} = 4.06\times10^{-6}m$

for $\lambda_{5\to6} = \frac{(4.14\times10^{-15})(3\times10^{8})}{0.166} = 7.48\times10^{-6}m$

5. a) $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.11\times10^{-31}(3\times10^{8})}(1-\cos 90) = 2.43\times10^{-12}m$$

(Note that h is in units of J·s)

b) The kinetic energy of recoiling electron is proportional to the change in energy of the photon

$$E_{photon\,initial} = \frac{hc}{\lambda_i} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2\times10^{-10}} = 6.21\times10^{3}eV$$

$$E_{photon\;final} = \frac{hc}{\lambda_f} = \frac{(4.14\times10^{-15})(3\times10^{8})}{2.0243\times10^{-10}} = 6.135\times10^{3}eV$$

(NOTE: h is in eV·s. Make sure you can convert from one form to the other and when you use it)

$$K_{electron} = -\Delta E_{photon} = 6.210\times10^{3} - 6.135\times10^{3} = 74.5eV$$

6. The kinetic energy of recoiling electron is proportional to the change in energy of the photon

$$K_{electron} = \frac{1}{2} m_e v^2 = \frac{1}{2}(9.11\times10^{-31})(1.5\times10^6)^2 = 1.02\times10^{-18}\,J$$

$$E_{photon\,initial} = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^8)}{8.5\times10^{-10}} = 2.34\times10^{-16}\,J$$

$$E_{photon\ final} = 2.34\times10^{-16} - 1.02\times10^{-18} = 2.33\times10^{-16}\,J \quad \text{(gave energy to electron)}$$

$$\lambda_f = \frac{hc}{E_{final}} = \frac{(6.63\times10^{-34})(3\times10^8)}{2.33\times10^{-16}} = 8.54\times10^{-10}\,m$$

b) To find the angle, we need to apply the Compton's scattering equation

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$

$$\theta = \cos^{-1}\left(1- [8.54\times10^{-10} - 8.5\times10^{-10}]\, (9.11\times10^{-31})(3\times10^{8})/ 6.63\times10^{-34}\right) = 130°$$

7. a) a photon travels at speed of light! So $v_{photon} = c = 3\text{x}10^8$ m/s

electron: $$p = mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}}{9.11\times10^{-31}(10^{-10})} = 7.3\times10^6\,m/s$$

baseball with estimate mass of 0.2 kg: $$v = \frac{h}{m_b\lambda} = \frac{6.63\times10^{-34}}{0.2(10^{-10})} = 3\times10^{-23}\,m/s$$

b) the energy of the photon is inversely proportional to its wavelength

$$E = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15})(3\times10^8)}{10^{-10}} = 1.2\times10^4\,eV$$

c) the kinetic energy of the electron:

$$K_{electron} = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31})(7.3\times10^6)^2 = 2.43\times10^{-17}\,J$$

$$K_{electron} = \frac{2.43\times10^{-17}\,J}{1.6\times10^{-19}\,J/eV} = 150eV$$

the kinetic energy of the baseball:

$$K_{baseball} = \frac{1}{2}mv^2 = \frac{1}{2}(0.2)(3\times10^{-23})^2 = 9\times10^{-47}\,J$$

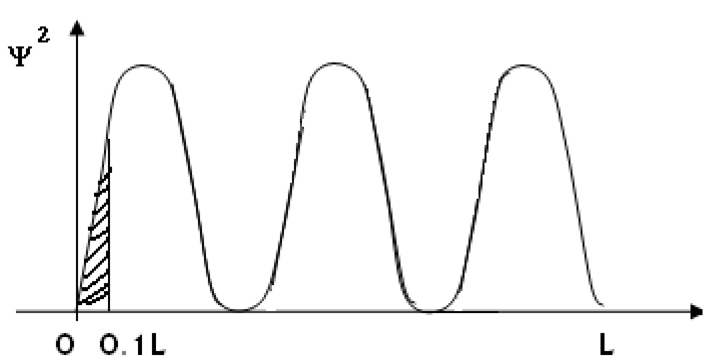
$$K_{baseball} = \frac{9\times10^{-47}\,J}{1.6\times10^{-19}\,J/eV} = 6\times10^{-28}eV$$

8. $E_{photon} = -\Delta E_{electron} = -(E_1 - E_2)$

$$E_{photon} = (2^2 - 1^2)\left(\frac{h^2}{8mL^2}\right) = \frac{3(6.63\times10^{-34})^2}{8(9.11\times10^{-31})(2.5\times10^{-9})^2} = 2.90\times10^{-20}\,J$$

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34})(3\times10^8)}{2.90\times10^{-20}} = 6.87\times10^{-6}m$$

9. a)



b) we first have to define the work function and the probability function

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\Psi^2(x) = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$$

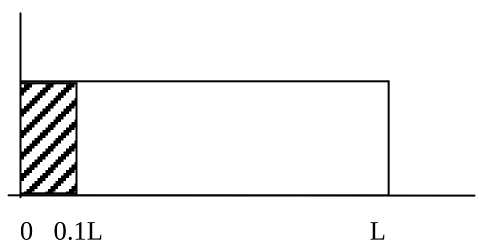
$$P_{0-0.1L} = \int_0^{0.1L} \Psi^2(x)dx$$ (corresponds to the area of shaded region)

$$P_{0-0.1L} = \frac{2}{L}\int_0^{0.1L} \sin^2\left(\frac{n\pi}{L}\right)xdx$$

$$P_{0-0.1L} = \frac{2}{L}\left[\frac{x}{2} - \frac{\sin 2\left(\frac{n\pi}{L}\right)x}{4\left(\frac{n\pi}{L}\right)}\right]_0^{0.1L}$$ with n = 3 (why? Explain!)

$$P_{0-0.1L} = \frac{2}{L}\left[\frac{0.1L}{2} - \frac{L\sin(0.6\pi)}{12\pi}\right] = 0.0495 = 4.95\%$$

c) Classically, equal probability everywhere in box. 1/10 chance of being between 0 and 0.1L



10. From the Heisenberg uncertainty principle for position and momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\Delta v \geq \frac{h}{4\pi \Delta x(m)} = \frac{6.63\times10^{-34}}{4\pi(1\times10^{-11})(1.67\times10^{-27})} = 3.15\times10^{3} m/s$$