Physics NYC

Problem set 5 diffraction

Part A

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C

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C

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A

Question 11
$\lambda_{sound} > \lambda_{visible\ light}$ so there is a much greater degree of diffraction

Question 12
The long axis should be vertical so that the sound is diffracted more horizontally (throughout the concert hall). Just like for single-slit diffraction of light, where a horizontal slit would diffract the light on the screen in a vertical pattern, and a vertical slit

would diffract the light on the screen in a horizontal pattern. This argument is only valid for relatively high frequencies overall.

Question 13

$d\sin\theta = m\lambda$ As the number of slits increases, the primary maxima get narrower

Part B

Question 1

$\lambda = 550$ nm
R = 2.50 m
y = 3.5 mm because (7 mm) / 2
$a\sin\theta = m\lambda$ is the formula for diffraction minima

$$\sin\theta = \frac{y}{R}$$

$$a\frac{y}{R} = m\lambda$$

m = 1 because we are dealing with the location of the first diffraction minima

$$a = \frac{\lambda R}{y} = \frac{550\times10^{-9}\times2.5}{3.5\times10^{-3}} = 3.93\times10^{-4}\text{ m}$$

Question 2

λ=630 nm
a = 0.15 mm
R = 3.00 m
a)
$a\sin\theta = m\lambda$ for diffraction minima

$$\sin\theta = \frac{y}{R}$$

$$a\frac{y}{R} = m\lambda$$

m = 1 because we wish to find the position of the first minimum

$$y = \frac{m\lambda R}{a} = \frac{1\times630\times10^{-9}\times3.00}{0.15\times10^{-3}} = 1.26\times10^{-2}\text{ m}$$

So the first diffraction minima is 1.26 cm from the center of the pattern

b)
the point halfway between the center of the pattern and the first intensity minimum is

$$\frac{1.26\text{x}10^{-2}}{2} = 6.3\text{x}10^{-3}\text{m}$$

NOTE: This solution uses the old textbook's definition of beta. You will get the same final answer if you use the OpenStax textbook's version of beta.

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \quad \text{with} \quad \sin\theta = \frac{y}{R}$$

We find $\beta = \pi$

$$I = I_0 \left(\frac{2}{\pi} \sin\frac{\pi}{2} \right)^2$$

$$I = 0.405 I_0$$

$I = 40.5\% \, I_0$

The intensity halfway to the first minimum is 40.5% of the intensity at the center.

Question 3

R = 4.00 m

$\lambda = 546 \times 10^{-9}$ m

From the picture, the first interference maximum is at 1 cm and the interference maximum at 4 cm is missing.

a)

$$\sin\theta = \frac{y}{R}$$

$d \sin\theta = m\lambda$ for interference maximum

m = 1 for the first minimum

$$d \frac{y}{R} = m\lambda$$

$$d = \frac{\lambda R}{y} = \frac{546 \times 10^{-9} \times 4.00}{1 \times 10^{-2}} = 2.18 \times 10^{-4} \text{ m}$$

b)

$a \sin\theta = m\lambda$ for diffraction minima

$$a \frac{y}{R} = m\lambda$$

$$a = \frac{m\lambda R}{y} = \frac{1 \times 5.46 \times 10^{-7} \times 4}{4 \times 10^{-2}} = 5.46 \times 10^{-5} \text{ m}$$

The width of the slit is 54.6 μm

Question 4

$\lambda = 630$ nm

R = 4.00 m

Central maximum to first minimum in single-slit diffraction $y = 3.15 \times 10^{-2}$ m

The distance between two successive interference maxima $y = 1.05 \times 10^{-2}$ m

a)

$a \sin\theta = m\lambda$ for diffraction minimum m = 1 for first minimum

$$\sin\theta = \frac{y}{R}$$

$$a = \frac{m\lambda R}{y} = \frac{630\times10^{-9}\times4.00}{3.15\times10^{-2}} = 8.00\times10^{-5}\text{ m}$$

The width of the slits is 80 μm

b)

$d\sin\theta = m\lambda$ for interference maximum

m = 1 for the first interference maximum

$$\sin\theta = \frac{y}{R}$$

$$d\frac{y}{R} = m\lambda$$

$$d = \frac{m\lambda R}{y} = \frac{6.3\times10^{-7}\times4}{1.05\times10^{-2}} = 2.40\times10^{-4}\text{ m}$$

The distance between the slits is 0.24 mm

c)

$a\sin\theta = 1\lambda$ first diffraction minimum

$d\sin\theta = m\lambda$ m^th interference maximum at same position (sinθ is the same for the diffraction minimum and the interference maximum)

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m\lambda}{1\lambda}$$

$$m = \frac{d}{a} = \frac{2.40\times10^{-4}}{8.00\times10^{-5}} = 3$$

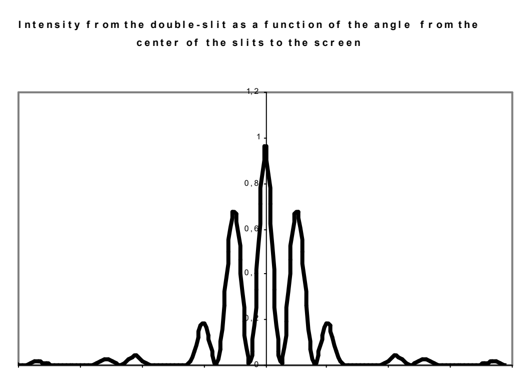
So the first missing maximum is 3.

d)

We have the central maximum (that's 1), then we have two maxima on each side (a total of 4) before the third on each side is missing.

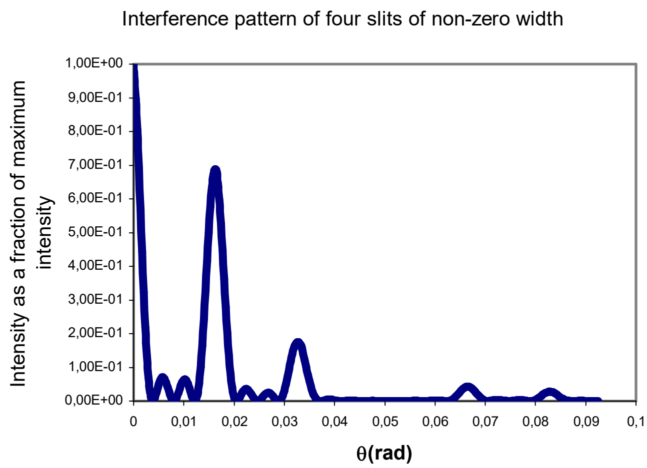
So the total number of interference maxima in the central maxima 5.

e)



Question 5

$\lambda = 500$ nm
d = 0.03 mm
a = 0.01 mm



a)

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

For the first interference maximum: $d\sin\theta = 1\lambda$

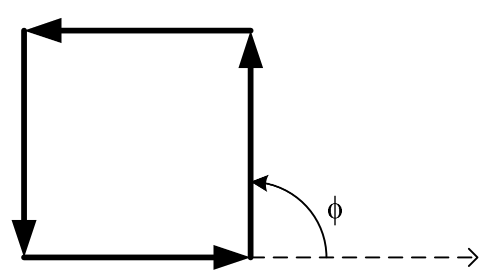
$$\phi = \frac{2\pi}{\lambda} \times 1\lambda = 2\pi$$

The phase angle (ϕ) of the first interference maximum is 2π.

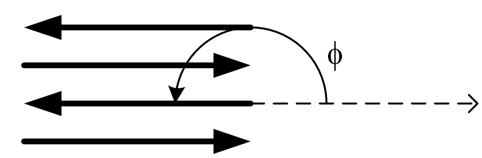
b)

There are N = 4 slits so we will find the minima at intervals of: $\frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$ or 90°

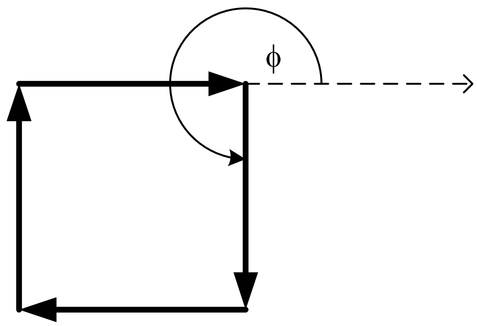
First minimum at $\phi = 90^{o}$:



Second minimum at $\phi = 180^{o}$



Third minimum at $\phi = 270^{o}$



c)
Intensity of the third principal maximum relative to the intensity of the central maximum.
We are at the third maximum so $\phi = 6\pi$

$I = I_0 \cos^2 \frac{\phi}{2}$ is the formula for intensity interference so the contribution from the interference effect is 1.

NOTE: This solution uses the old textbook's definition of beta. You will get the same final answer if you use the OpenStax textbook's version of beta.

$I = I_0 \left[\frac{\sin \frac{\beta}{2}}{\frac{\beta}{2}} \right]^2$ is the formula for intensity in diffraction

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$d \sin\theta = m\lambda \rightarrow \sin\theta = \frac{m\lambda}{d} = \frac{3\lambda}{d}$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \rightarrow \beta = \frac{2\pi}{\lambda} a \frac{3\lambda}{d} = 2\pi \frac{3\times a}{d} = 2\pi \frac{3\times 0.01\times 10^{-3}}{0.03\times 10^{-3}} = 2\pi$$

$$I = I_0 \left[\frac{\sin\pi}{\pi} \right]^2 = 0$$

The third principal maximum is a missing maximum as can be seen from the diagram at the beginning of the solution

d)
$I=N^2I_0$
The intensity of the central maximum is 16 times as bright ($4^2=16$) as the central maximum of a 1-slit pattern.

Question 6

Diffraction grating 350 lines per mm $\frac{350}{10^{-3}} = 350000$ lines per m

$$d = \frac{1}{350000} = 2.83 \times 10^{-6}\,\text{m}$$

Visible spectrum is 400-700nm.

a)

We must find the angular width of the first order i.e. m=1

$$d \sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

For 400 nm : $\theta = \sin^{-1}\left(\frac{1 \times 400 \times 10^{-9}}{2.86 \times 10^{-6}}\right) = 8.04°$

For 700 nm: $\theta = \sin^{-1}\left(\frac{1 \times 700 \times 10^{-9}}{2.86 \times 10^{-6}}\right) = 14.17°$

Angular width: 14.17°-8.04°=6.13°

The angular width of the first order is 6.13°

b)

We must find the angular width of the 3rd order i.e. m=3

For 400 nm : $\theta = \sin^{-1}\left(\frac{3 \times 400 \times 10^{-9}}{2.86 \times 10^{-6}}\right) = 24.8°$

For 700 nm: $\theta = \sin^{-1}\left(\frac{3 \times 700 \times 10^{-9}}{2.86 \times 10^{-6}}\right) = 47.2°$

Angular width: 24.8°-47.2°=22.4°

The angular width of the first order is 22.4°

Question 7

This is an interference problem from reflected rays, which obeys the same principle as interference through slits.

$$d \sin\theta = m\lambda$$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

m = 1 $\quad \theta = \sin^{-1}\left(\frac{1 \times 632.8 \times 10^{-9}}{1.6 \times 10^{-6}}\right) = 23.3°$

m = 2 $\quad \theta = \sin^{-1}\left(\frac{2 \times 632.8 \times 10^{-9}}{1.6 \times 10^{-6}}\right) = 52.3°$

m=3 does not exist

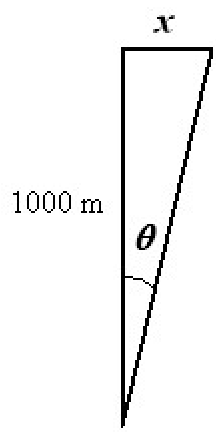
the intensity of the reflected light will be maximum at 0°, 23.3°, and 52.3°.

Question 8

a) ASSUMPTION: Point sources
ASSUMPTION: Ideal conditions (e.g. no 'twinkling')
ASSUMPTION: Take pupil diameter as 3 mm
ASSUMPTION: Use $\lambda = 550$ nm (approximate centre of visible spectrum)

$$\theta = 1.22\frac{\lambda}{D}$$

$$= 1.22\frac{5.5\times10^{-7}}{3\times10^{-3}} = 2.2\times10^{-4} \text{ rad}$$



$$x = 1000\tan\theta$$

$$= 0.22 \text{ m}$$

b) $$\theta = 1.22\frac{5.5\times10^{-7}}{0.1} = 6.71\times10^{-6} \text{ rad}$$

$$x = 1000\tan\theta$$

$$= 6.7\times10^{-3} \text{ m}$$

c) The wavelength of light is smaller inside the eye $\left(\lambda_{\text{eye}} = \frac{\lambda}{n}\right)$.

Therefore θ is smaller $\left(\theta = 1.22\frac{\lambda}{D}\right)$ and x would also be smaller.

N.B. Make sure you understand the geometry of the above diagram! Study the diagram on the next page.

Consider light coming from source A moving toward the bottom of the page, and entering the eye through the pupil (diagram definitely not to scale!):