Physics NYC

Problem set 4 interference

Part 1:

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A

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A

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D

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B

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B

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A

Question 11
A laser source produces coherent light (i.e. all produced waves have a constant phase difference among each other) while an ordinary light source produces incoherent light. The phase difference between all waves emitted by an ordinary light source changing all the time means that it is impossible to build constant constructive and destructive interference patterns.

A laser produces monochromatic light, which makes it easier to see the interference pattern. An ordinary light source produces all visible wavelengths; if interference was possible, the interference patterns of the different colours would overlap, making it difficult to distinguish the individual monochromatic patterns.

Question 12

The index of refraction n of a material is the ration of the speed of light in vacuum to the speed of light in the medium. For water, the index of refraction is 1.3, meaning the speed of light is 1.3 times smaller in water than the speed of light in vacuum (or air.)

The speed of a wave, its wavelength and its frequency are related by $v = \lambda f$. Since the frequency does not change when the light changes medium, if the speed of light deceases, its wavelength decreases as well.

The distance y of each interference fringe on the screen with repect to the central bright one is given by $y=m\lambda L/d$ where m is the interference order, and d is the slit separation. If m, L and d remain constant, one sees that if λ decreases, the y decreases.

The value y gets smaller, so the interference pattern is compressed when the experiment is done underwater.

Question 13

A fraction of the light incident on the oil film is directly reflected and another fraction goes through the oil film to be reflected back through the oil film by the water surface. A fraction of this reflected light will emerge into air. This last emergent light will interfere with the light that was initially reflected by the oil surface.

When a ray in medium I is reflected at the interface between medium I and medium II, there is no phase shift if the index of refraction of II is smaller than that of I (soft reflection). On the other hand, if the index of refraction of II is greater than that of I, the ray will undergo a phase shift of π radians upon reflection (hard reflection).

The second reflected ray could have a phase shift with the first one due to the fact that there is a path difference travelling back and forth through the thickness of the oil film. At the edge of the film, this thickness is small so the second reflected ray has no phase shift due to the path: π.

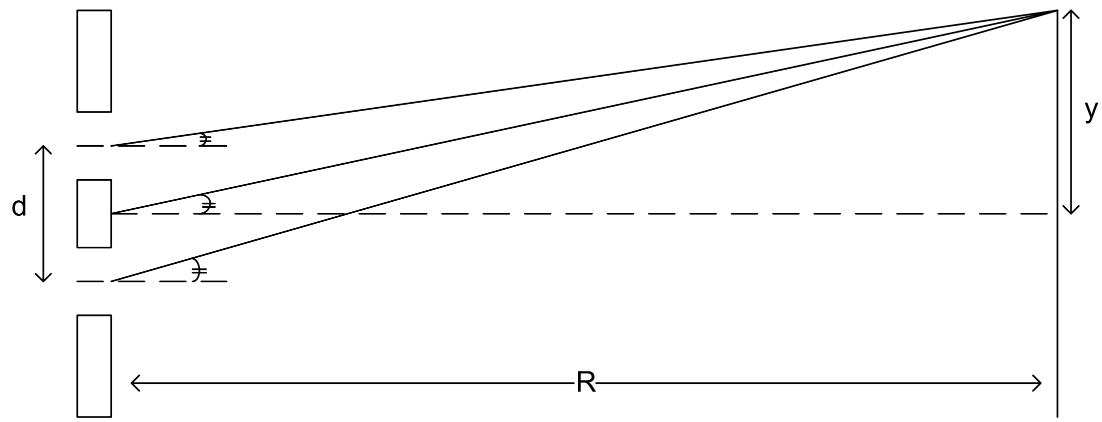
In order to have the brightest reflection, both reflected rays must be in phase (or $\phi_{net} = \phi_{reflection} + \phi_{path} = 0, 2\pi, 4\pi$, etc.). Since $\phi_{path} = 0$, $\phi_{reflection}$ must also be 0. The ray at the air-oil interface undergoes a phase shift of π ($n_{oil} > n_{air}$), so there must also be a phase shift of π for the ray reflecting from the oil-water interface. Therefore, n_{water} must be greater than n_{oil}.

Question 14

B

Part B

Question 1



a)

By the small angle approximation, all angles θ in the picture are the same.
y = 3 cm (from the center of the pattern to m = 1 maximum)
λ = 589 nm
R = 1.50 m

$$d\sin\theta = m\lambda$$

by the small angle approximation: $\sin\theta = \dfrac{y}{R}$

replace sinθ in the first expression to get: $d = \dfrac{m\lambda R}{y}$

$$d = \frac{1\times 5.89\times 10^{-7}\times 1.5}{3\times 10^{-2}} = 2.95\times 10^{-5}\ \text{m}$$

The distance between the slits is 29.5 μm

b)

y = 1×10^{-2} m (distance from an intensity maximum)

$$\phi = \frac{2\pi}{\lambda} d\sin\theta$$

by the small angle approximation: $\sin\theta = \dfrac{y}{R}$

replace sinθ in the first expression to get: $\phi = \dfrac{2\pi}{\lambda} d \dfrac{y}{R}$

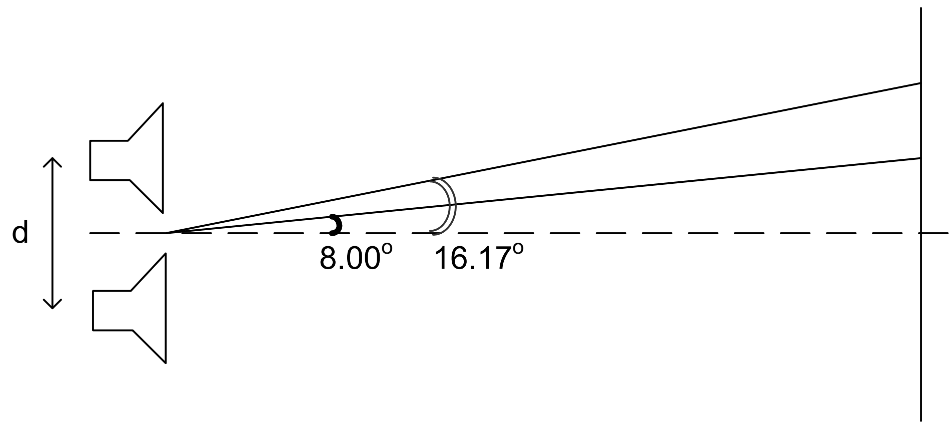
$$\phi = \frac{2\pi}{5.89\times 10^{-7}} 2.95\times 10^{-5} \frac{1\times 10^{-2}}{1.5} = 2.10\ \text{rad}$$

$$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{2.10}{2}\right)$$

$$\frac{I}{I_{max}} = 0.248$$

The intensity is ¼ of the maximum intensity 1 cm from an intensity maximum

Question 2



a)
d = 2 m
$d \sin\theta = m\lambda$ for constructive interference

m = 1 $\lambda = \frac{d \sin\theta}{m}$

$$\lambda = \frac{2\sin(8°)}{1} = 0.278 \text{ m}$$

m = 2 λ = 2sin(16.17°)/2 = 0.278 m (same, as expected)

The wavelength of the sound waves is λ = 0.278 m

b)

f = v/λ = 343/0.278 = 1234 Hz

The frequency of the sound wave is 1234 Hz

c)
m = 3 $d \sin\theta = m\lambda$

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times 0.28}{2}\right) = 24.8°$$

In the same way, we find that constructive interference is heard at the following angles:
m = 4 $\theta = 34°$

m = 5 $\theta = 44.4°$
m = 6 $\theta = 57.1°$
m = 7 $\theta = 78.5°$

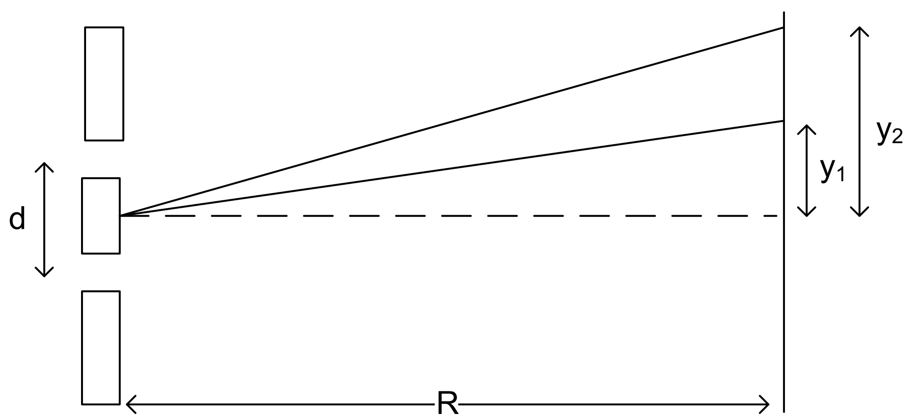
m = 8 does not exist, because $\frac{8 \times 0.28}{2}$=1.12 and there is no such thing as a sine bigger than 1.

d)
We wish to find the smallest angle at which the sound waves completely cancel.
We choose m = 0 for the smallest angle
We use: $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ for destructive interference

$$\theta = \sin^{-1}\left(\left(0 + \frac{1}{2}\right) \times \frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{0.28}{2 \times 2}\right) = 4.01°$$

The smallest angle at which the sound waves completely cancel is 4.01°

Question 3



d = 100λ
R = 50 cm
$d \sin\theta = m\lambda$ for constructive interference

by the small angle approximation: $\sin\theta = \frac{y}{R}$

replace sinθ in the first expression to get: $y = \frac{m\lambda R}{d}$

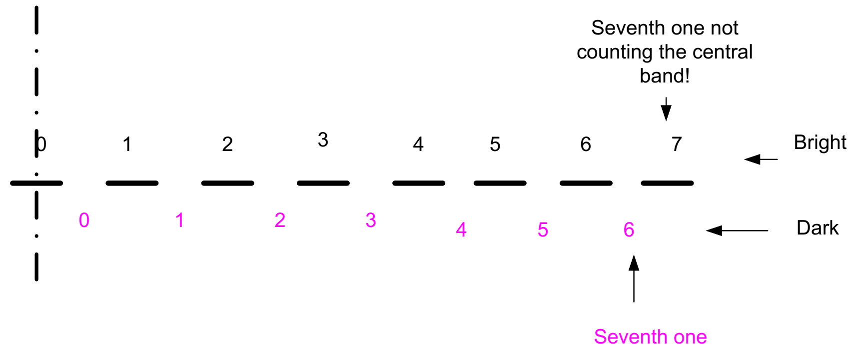
First maximum: m = 1 $y_1 = \frac{1 \times \lambda R}{d}$

Second maximum: m = 2 $y_2 = \frac{2 \times \lambda R}{d}$

$$y_2 - y_1 = \Delta y = \frac{\lambda R}{d} = \frac{\lambda \times 0.5}{100\lambda} = 5 \times 10^{-3} m$$

The distance between the two maxima is 5 mm

Question 4



$d\sin\theta = m_1\lambda_1$ (constructive interference) m=7 for the seventh bright fringe, λ_1=540 nm

$d\sin\theta = \left(m_2 + \frac{1}{2}\right)\lambda_2$ (destructive interference), m=6 for the seventh dark fringe, we are looking for λ_2

In both cases, the slits used are the same so d is the same in both cases.
Also, the fringes are at the same position so sinθ is the same in both cases.

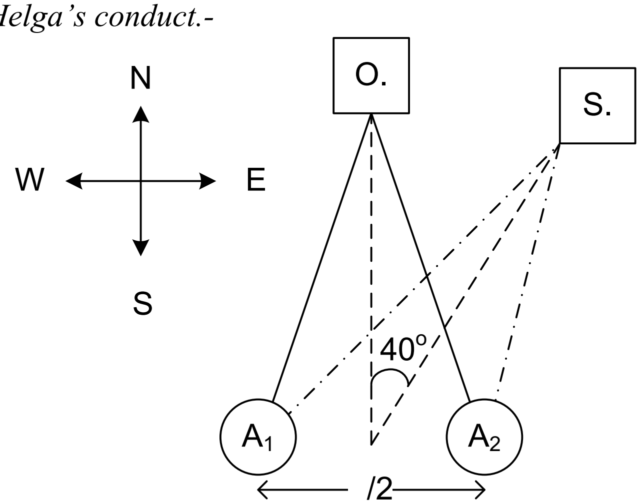
$$m_1\lambda_1 = \left(m_2 + \frac{1}{2}\right)\lambda_2$$

$$\lambda_2 = \frac{m_1\lambda_1}{m_2 + \frac{1}{2}} = \frac{7\times 540\times 10^{-9}}{6 + \frac{1}{2}} = 582\times 10^{-9}\ \text{m}$$

The second light has a wavelength of 582 nm

Question 5

-Please note that the Physics Department of John Abbott College does not approve of Helga's conduct.-



$d = \frac{\lambda}{2}$ (d is the distance between two sources, whether light sources or antennae

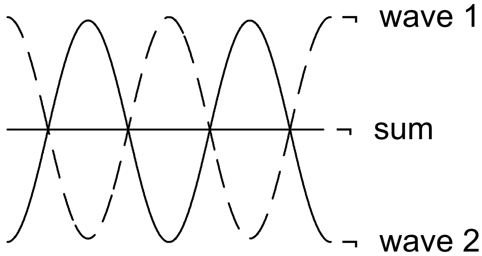
θ = 0 for Orville

θ = 40° or 0.7 rad for Siegfried

total phase difference = phase difference from antenna + phase difference from paths

$$\phi_{TOT} = \phi_a + \frac{2\pi}{\lambda} d \sin\theta$$

If two waves are out of phase by 180°, or π radians, they will cancel exactly as can be seen in the picture below:



Therefore we want the total phase difference to be 180° or π rad.

We want the signal arriving at Siegfried to be zero

$$\phi_{TOT} = \phi_a + \frac{2\pi}{\lambda} d \sin\theta = \pi$$

$$\phi_a = \pi - \frac{2\pi}{\lambda} d \sin\theta$$

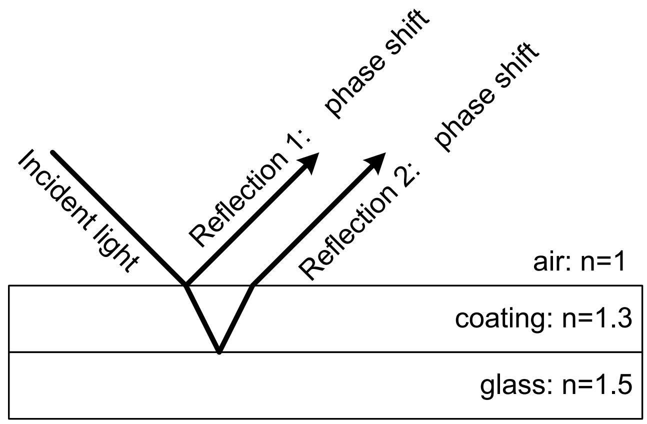
$$\phi_a = \pi - \frac{2\pi}{\lambda}\frac{\lambda}{2}\sin(0.7) \text{ or } \phi_a = \pi - \frac{2\pi}{\lambda}\frac{\lambda}{2}\sin(40°)$$

$$\phi_a = 1.12 \text{ rad or } 64°$$

The phase difference between the antennae should be set at 1.12 rad or 64°

Question 6

λ = 500 nm



We have 2 phase shifts of π which cancel so the formula to use for destructive interference is:

$$2t = \left(m + \frac{1}{2}\right)\lambda$$

The light is travelling through a material (the coating), so λ is the wavelength in the material:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n}$$ where λ_0 is the wavelength in air.

We must use m = 0 because the problem asks for the **minimum** thickness.

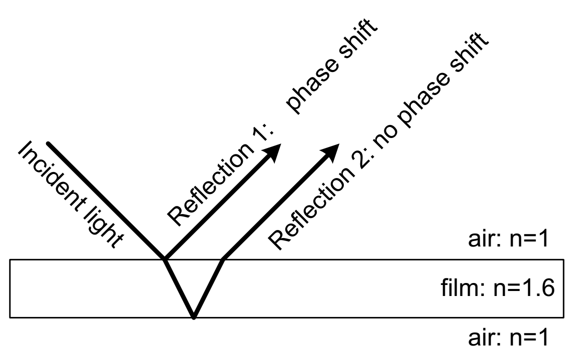
$$t = \frac{1}{2}\left(0 + \frac{1}{2}\right)\frac{500\times10^{-9}}{1.30}$$

$t = 96.1 \times 10^{-9}$ m

The coating must have a thickness of 96.1 nm.

Question 7

$\lambda_1 = 504$ nm
$\lambda_2 = 672$ nm (red)



a)
We have **one phase shift of π** in one reflection and **no phase shift** for the other reflection
In this case the formula for **destructive** interference is:

$2t = m\lambda$

The light is travelling through a material (the film), so λ is the wavelength in the material:

$2t = m\frac{\lambda_0}{n}$ where λ_0 is the wavelength in air.

The problem here is that **we don't know m** (yet). The problem asks for the **minimum** thickness, but there are **two wavelengths** for which the condition must be met.

Let's call m_1 the order for λ_1 that is 504 nm.

The only other missing wavelength is λ_2 = 672 nm. We'll call m_2 the order for λ_2.

λ_2 is longer than λ_1 so if the two wavelengths go through the same film (same thickness t), it makes sense that $m_2 < m_1$. (Make sure you understand this before you continue.)

Here comes the math:

$$2t = m_1\frac{\lambda_1}{1.6} = m_2\frac{\lambda_2}{1.6}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{m_1}{m_2}$$

$$\frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{672}{504} = 1.3333 \text{ or } \frac{4}{3}$$

$m_1 = 4$ is bigger than $m_2 = 3$ as predicted (phew) (note that $m_1 = 8$ and $m_2 = 6$ would also work but would not yield the thinnest thickness)

We can now use either m_1 and λ_1 or m_2 and λ_2 to find the thickness of the film.

$$2t = m\frac{\lambda_0}{n}$$

$$t = \frac{1}{2}\times 4\times\frac{504\times10^{-9}}{1.6} = 630 \text{ nm}$$

The thickness of the film is 630 nm.

b)

We now wish to find the most strongly reflected wavelengths. We have one phase shift of π in one reflection and no phase shift for the other reflection

In this case the formula for **constructive** interference is:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n}$$

$$\lambda_0 = \frac{2tn}{\left(m + \frac{1}{2}\right)}$$

Try: m = 1 λ_0 = 1344 nm not visible (infrared)

m = 2 λ_0 = 806.4 nm not visible (near infrared)

m = 3 λ_0 = 576 nm visible

m = 4 λ_0 = 448 nm visible

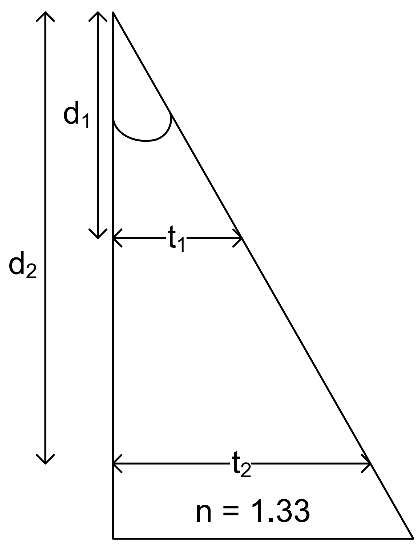
m = 5 λ_0 = 366.5 nm not visible (ultraviolet)

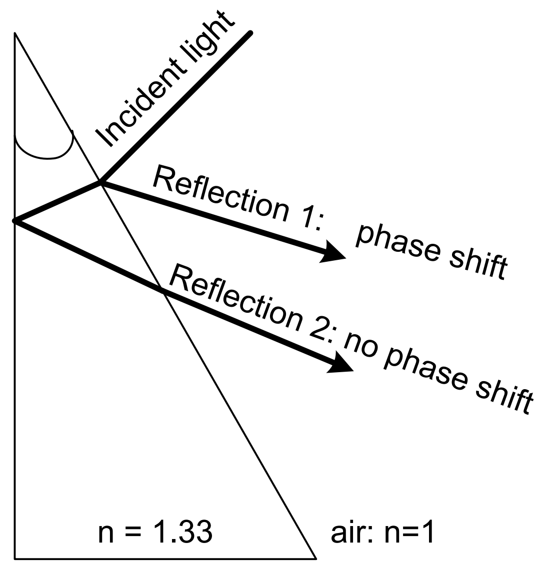
The wavelengths most strongly reflected are 576 nm and 448 nm.

Question 8

First some vocabulary:
A **wedge** is a piece of metal, wood, rubber, etc. with a pointed edge at one end and a wide edge at the other, which is either pushed between two objects to keep them still or forced into something to break pieces off it. (**French: coin; German: der Keil**)

Then, the given and the diagram:
$\lambda_1 = 425$ nm
$\lambda_2 = 680$ nm
$d_1 = 1.20$ cm





We have one phase shift of π in one reflection and no phase shift for the other reflection, there the formula for constructive interference becomes.

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n}$$

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{2n}$$

$$\tan\theta = \frac{t_1}{d_1} \qquad t_1 = d_1 \tan\theta$$

$$\tan\theta = \frac{t_2}{d_2} \qquad t_2 = d_2 \tan\theta$$

$$d_1 \tan\theta = \left(m + \frac{1}{2}\right)\frac{\lambda_1}{2n} \qquad d_2 \tan\theta = \left(m + \frac{1}{2}\right)\frac{\lambda_2}{2n}$$

$$\frac{d_2 \tan\theta}{d_1 \tan\theta} = \frac{\left(m+\frac{1}{2}\right)\frac{\lambda_2}{2n}}{\left(m+\frac{1}{2}\right)\frac{\lambda_1}{2n}}$$

$$\frac{d_2}{d_1} = \frac{\lambda_2}{\lambda_1}$$

$$d_2 = \frac{\lambda_2}{\lambda_1} d_1$$

$$d_2 = \frac{680}{425} x 1.2x10^{-2} = 1.92x10^{-2}\, m$$

The first bright fringe for 680 nm is from the top of the wedge 1.92 cm.

b)

Position of second bright fringe for 425 nm.

For the first bright fringe m = 0: $2t_{\text{first}} = \left(0+\frac{1}{2}\right)\frac{425\times10^{-9}}{n}$

For the second bright fringe m = 1: $2t_{\text{second}} = \left(1+\frac{1}{2}\right)\frac{425\times10^{-9}}{n}$

So we apply the same technique as before: $\frac{2t_{\text{second}}}{2t_{\text{first}}} = \frac{\left(1+\frac{1}{2}\right)\frac{425\times10^{-9}}{n}}{\left(0+\frac{1}{2}\right)\frac{425\times10^{-9}}{n}}$

$$t_{\text{second}} = \frac{3}{2}\times\frac{2}{1}\times t_{\text{first}} = 3\times1.2\times10^{-2} = 3.6\times10^{-2}\text{ m}$$

The second bright fringe is 3.6 cm down the wedge.