

PROBLEM SET 1
SIMPLE HARMONIC MOTION

Part A

1. C
2. C
3. D
4. D
5. E
6. B
7. D
8. C
9. B
10. D

11. Object in *second quarter* of motion → moving toward maximum displacement

- Kinetic energy decreasing
- Moving away from equilibrium
- v greatest when $x = 0$
- at $x = -2.5$ (which is $-A$)

12. **period (T)** no change

k: no change

E_{tot} : four times

v_{max} : two times

a_{max} : two times

ω: no change

13. a) T increases (like being on planets with smaller g)

b) T decreases (like being on planets with larger g)

c) no change

Part B

1. a) the angular frequency and frequency are both related together by the following equation:

$$\omega = 2\pi f = 12.56 \text{ rad/s} \qquad f = \frac{12.56}{2\pi} = 2.00 \text{ Hz}$$

b) The maximum velocity: $v_{\max} = \omega A = 12.56 \times 15.0 = 188 \text{ cm/s}$

c) From the simple harmonic equation of the wave, we can find the velocity equation as a function of time

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$v = -(12.56)(15) \underbrace{\sin(12.56 \times 0.25 + \frac{\pi}{2})}_{=-1}$$

$$v = +188 \text{ cm/s}$$

$t = 0.25 \text{ s}$ is $\frac{1}{2} T$

object is at phase $\frac{3\pi}{2}$ rad

d) Because the velocity is max, the position and the acceleration will have to be min at that time.

$$a = 0; \text{ again, solve by inspection or plug into } a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

e). From the angular frequency equation, we can obtain the spring constant

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m = (12.56)^2 \times 0.2 = 31.6 \text{ N/m}$$

2. a) The total energy of the system is given by:

$$E_{\text{tot}} = \frac{1}{2} k A^2 \quad \text{with the spring constant } k = \omega^2 m = \left(\frac{\pi}{3}\right)^2 \times 3 = 3.29 \text{ N/m}$$

$$E_{\text{tot}} = \frac{1}{2} (3.29)(0.5)^2 = 0.411 \text{ J}$$

b) To find the potential energy of the system, we have to find the position at a specific time.

$$x = 0.5 \cos\left(\frac{\pi}{3} \times 1.3 - \frac{\pi}{4}\right) = 0.419m$$

$$U_{elastic} = \frac{1}{2} kx^2 = \frac{1}{2} (3.29)(0.419)^2 = 0.289J$$

c) The total energy is the sum of the kinetic and potential energy

$$K + U_{elastic} = E_{tot} \Rightarrow K = E_{tot} - U_{elastic}$$

$$K = 0.411 - 0.289 = 0.122J$$

d) When the potential energy is half the total energy ($U_{elastic} = \frac{1}{2} E_{tot}$)

$$\frac{1}{2} kx^2 = \frac{1}{2} \left(\frac{1}{2} kA^2\right)$$

$$x^2 = \frac{1}{2} A^2 \Rightarrow x = \frac{A}{\sqrt{2}} = 0.354m$$

3. a) The period and the frequency are both related together by the following equation:

$$T = \frac{1}{f} = \frac{1}{0.25} = 4s$$

b) The angular frequency:

$$\omega = 2\pi f = 2\pi(0.25) = \frac{\pi}{2}$$

c) The total energy is the sum of the potential energy and kinetic energy

$$E_{total} = U_{elastic} + K$$

$$E = \frac{1}{2} kx^2 + \frac{1}{2} mv^2$$

$$\text{with } \omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m = \left(\frac{\pi}{2}\right)^2 (0.1) = 0.247N/m$$

$$E = \frac{1}{2} (0.247)(2.8 \times 10^{-3})^2 + \frac{1}{2} (0.1)(4 \times 10^{-3})^2 = 1.77 \times 10^{-6} J$$

d) To can obtain the amplitude of motion with the total energy

$$E_{total} = \frac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{2E_{total}}{k}} = \sqrt{\frac{2(1.77 \times 10^{-6})}{0.247}} = 3.79 \times 10^{-3} m$$

e) The maximum velocity can be obtain from the maximum kinetic energy equation or from the maximum velocity equation

$$K_{max} = E_{total} = \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{2E_{total}}{m}}$$

$$v_{max} = \sqrt{\frac{2(1.77 \times 10^{-6})}{0.1}} = 5.95 \times 10^{-3} m/s$$

$$\text{or } v_{max} = \omega A = \left(\frac{\pi}{2}\right)(3.79 \times 10^{-3}) = 5.95 \times 10^{-3} m/s$$

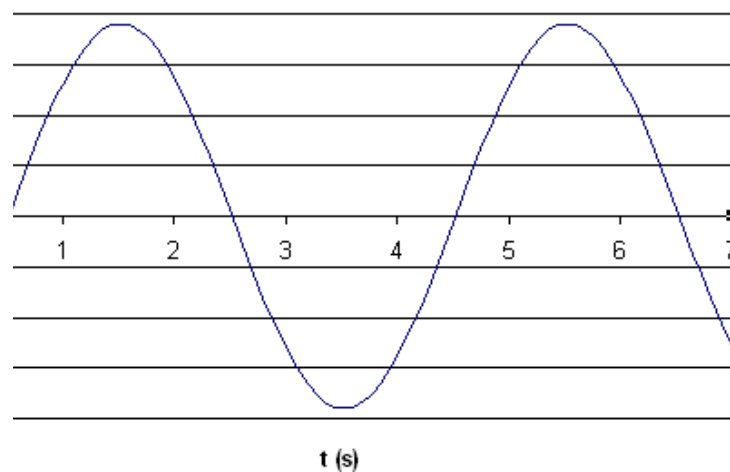
f) The maximum acceleration is found from the equation of the acceleration as a function of time

$$a_{max} = \omega^2 A$$

$$a_{max} = \left(\frac{\pi}{2}\right)^2 (3.79 \times 10^{-3}) = 9.35 \times 10^{-3} m/s^2$$

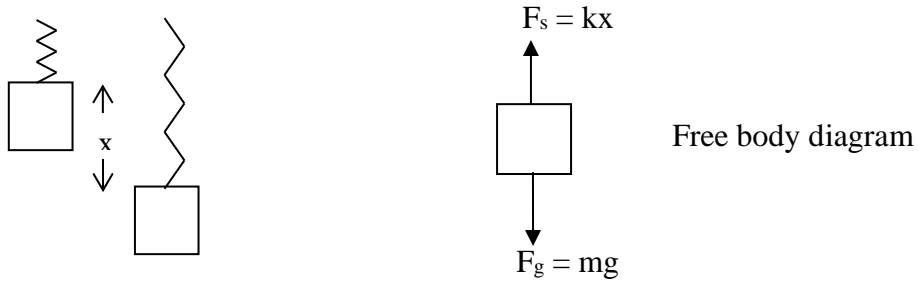
$$\text{or } a = \frac{kx}{m} \quad a_{max} = \frac{kA}{m}$$

g)

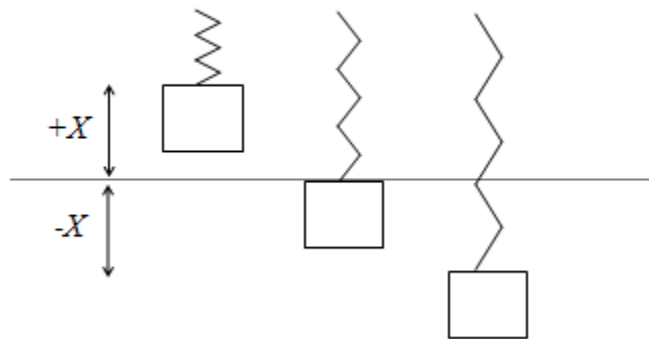


4. a) At equilibrium position,

$$F_s = F_g \Rightarrow x = \frac{mg}{k} = \frac{2(9.81)}{150} = 0.131m$$



b) Amplitude $A = 0.131$ m, therefore, it will fall 0.262 m below initial position.



c) To find the position as a function of time, we need to find the phase and the angular frequency.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{2}} = 8.66 \text{ rad/s}$$

$$\phi = 0 \quad (\text{starts at } +A \text{ at } t = 0)$$

$$A = 0.131 \text{ m}$$

$$y = 0.131 \cos(8.66t + 0)$$

5. From the position equation, we can obtain the acceleration equation as a function of time

$$x = A \cos(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \underbrace{\cos(\omega t + \phi)}_{=\pm 1}$$

$$a_{\max} = \omega^2 A \quad \text{with} \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{1.7} = 3.70 \text{ rad/s}$$

$$a_{\max} = (3.70)^2 (0.25) = 3.42 \text{ m/s}^2$$

The maximum force

$$F_{\max} = \mu N = ma$$

$$\mu mg = ma \Rightarrow \mu = \frac{a}{g} = \frac{3.42}{9.81} = 0.349$$

6. a) The period of oscillation depends on the length of the string and the gravity.

$$T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 (2.3)}{(4.25)^2} = 5.03 \text{ m/s}^2$$

b) The angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.25} = 1.48 \text{ rad/s}$$

c) The angular position equation is obtained by finding the phase, the angular frequency and the amplitude.

$$\phi = 0 \quad \omega = 1.48 \text{ rad/s} \quad A = 12^\circ$$

$$\theta = (12^\circ) \cos(1.48t + 0)$$

d) At $t = 1.5$, the angular position of the pendulum is:

$$\theta = (12^\circ) \cos(1.48 \times 1.5) = -7.25^\circ$$

e) The angular velocity can be obtained by taking the derivative as a function of time of the angular position equation (note that ω_z represents the angular velocity and ω represents the angular frequency... which of course are not the same!)

$$\omega_z = \frac{d\theta}{dt} = -\omega A \sin(\omega t + \phi)$$

$$\omega_z = -1.48 \times 12^\circ \sin(1.48 \times 1.5 + 0) = -14.1^\circ / s$$

7. a) The amplitude is obtained by finding the total energy when the velocity is max

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2 \Rightarrow A = \sqrt{\frac{mv_{\max}^2}{k}} = \sqrt{\frac{(0.3)(2)^2}{12}} = 0.316m$$

You could also use the equation for the mass' maximum velocity ($v_{\max} = \omega A$), noting that this equals the elevator's velocity (since this is the velocity the mass has when the elevator stops).

$$\begin{aligned} v_{\max} &= \omega A \\ 2 &= (k/m)^{1/2} A \\ 2 &= (12/0.3)^{1/2} A \\ 2 &= 6.32A \\ A &= 0.316 \text{ m} \end{aligned}$$

b) the position equation depends on the angular frequency, the amplitude and the phase

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{0.3}} = 6.32 \text{ rad} / s$$

$$\phi = \frac{\pi}{2} \text{ (at } t = 0, \text{ it is at equilibrium and moving downward)}$$

$$y = 0.316 \cos(6.32t + \frac{\pi}{2})$$

8. If plate acceleration is greater than g downward, then the block will not accelerate quickly enough to keep contact with it. The only forces on block are the normal N and the weight and N can't be less than 0.

$$\omega = 2\pi f = 4\pi \text{rad} / \text{s}$$

$$a_{\max} = \omega^2 A \Rightarrow A = \frac{9.81}{(4\pi)^2} = 0.0621\text{m} = 6.21\text{cm}$$

9. a) The angular frequency is found from the period: $T = 0.06\text{s}$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.06} = 105\text{rad} / \text{s}$

b) $A = 0.06\text{m}$

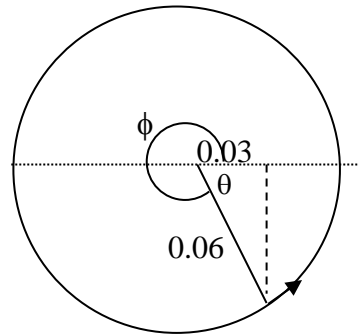
ϕ : For a cosine function, it is $5/6$ of the way through its cycle. At $t = 0$, $\phi = \frac{5}{6}(2\pi) = 5.24\text{rad}$

or

$$\theta = \cos^{-1}\left(\frac{0.03}{0.06}\right) = 1.047\text{rad}$$

$$\phi = 2\pi - \theta = 5.24\text{rad}$$

$$x = 0.06\cos(105t + 5.24)$$



c) at $t = 0.1\text{s}$, the position is:

$$x = 0.06 \cos(10.5 + 5.24) = -0.06 \text{ m}$$

Also by inspection: time is $1(T)$ after 0.04s .

d) From the acceleration equation, we can find the maximum acceleration. When ...

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \underbrace{\cos(\omega t + \phi)}_{=1}$$

$$a = -105^2(0.06) = -662\text{m}/\text{s}^2$$

e) The spring constant is found from the angular frequency

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m = (105)^2 (4) = 4.41 \times 10^4 \text{ N/m}$$

or from the force equation of a spring

$$k = \frac{F}{x} = \frac{ma}{x} = \frac{4(662)}{0.06} = 4.41 \times 10^4 \text{ N/m}$$