Supplementary Problems Set 3

1. Given that the magnetic force experience by an electron will be given by $\vec{F_B} = q\vec{v} \times \vec{B}$ we will have to take the determinant approach to calculating the force. Using:

$$\begin{vmatrix} i & j & k \\ 3 & -2 & 0 \\ -1.2 & 0 & 4 \end{vmatrix}$$

we obtain that the result of this calculation $\vec{v} \times \vec{B} = \left(8 - \hat{\imath} + 12 - \hat{\jmath} + 2.4 - \hat{k}\right) \cdot 10^5$. Let us just remember to multiply by the charge AND sign of the electron to obtain the final answer $\boxed{\vec{F_B} = \left(1.3\hat{\imath} + 1.9\hat{\jmath} + 0.38\hat{k}\right) \cdot 10^{-13}N}$

2. This is a classical mass spectrometer question. Let us therefore use conservation of energy to figure out the velocity with which it exits the acceleration plates:

$$qV = \frac{mv^2}{2} \rightarrow v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 2000}{24 \cdot 1.67 \cdot 10^{-27}}} = 1.27 \cdot 10^5 m/s \tag{1}$$

Now that we have the velocity, we may use the formula from the formula sheet to figure out the radius:

$$r = \frac{mv}{qB} = \frac{24 \cdot 1.67 \cdot 10^{-27} \cdot 1.27 \cdot 10^5}{1.6 \cdot 10^{-19} \cdot 0.05} = 0.64m \tag{2}$$

Therefore, the radius of cuvrature is $\boxed{0.64m}$

3. (a) We know that the simplest way to obtain the velocity of a charged particle in a velocity selector is to use the $v = \frac{E}{B}$ formula. We are given B and we can find E by using $V = Ed$.

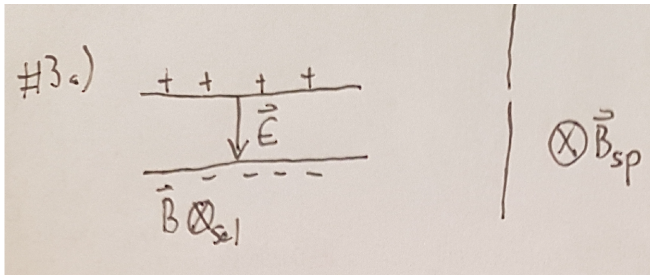
$$E = \frac{V}{d} = \frac{1.6 \cdot 10^2}{2 \cdot 10^{-3}} = 8 \cdot 10^4 \tag{3}$$

Therefore the velocity is $\boxed{v = \frac{8 \cdot 10^4}{0.42} = 1.9 \cdot 10^5 m/s}$

You can see the relative directions of fields in the following diagram:

(b) To find the separation between the two peaks of the two different isotopes, we will need to use the formula that gives us radius of curvature as a function of all the parameters of the velocity selector: $r = \frac{mv}{qB}$

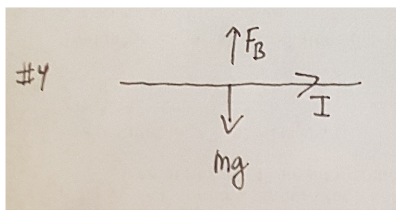
$$r = \frac{mv}{qB} = \frac{24 \cdot 1.67 \cdot 10^{-27} \cdot 1.9 \cdot 10^5}{1.6 \cdot 10^{-19} \cdot 1.2} = 4.0cm \tag{4}$$



Using the same equation with the mass being 26 yields a radius of 4.3 cm.

Therefore the difference in positions where they will hit the screen is given by $\boxed{2r_{26} - 2r_{24} = 6mm}$

4. We are looking for the current and we are given the length, the force and the magnetic field. We can just plug everything into $F = ILB$ since everything is perpendicular as can be seen from the picture:



$$F = ILB \rightarrow I = \frac{F}{LB} = \frac{0.24}{0.2 \cdot 0.5} = 2.4 \tag{5}$$

Therefore the current in the wire is $\boxed{2.4A}$

5. We know that torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$ so we must calculate the magnitude and direction of the torque. Let us begin with the magnitude.

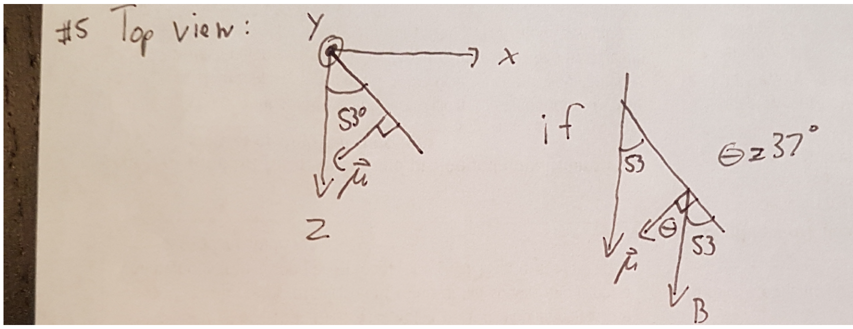
$$\tau = \mu \times B = NIABsin(\theta) = 300 \cdot 0.2 \cdot 0.05 \cdot 0.08 \cdot 0.5 \cdot sin(37) = 0.072Nm \tag{6}$$

Following the picture can give us a better idea of the direction of everything:

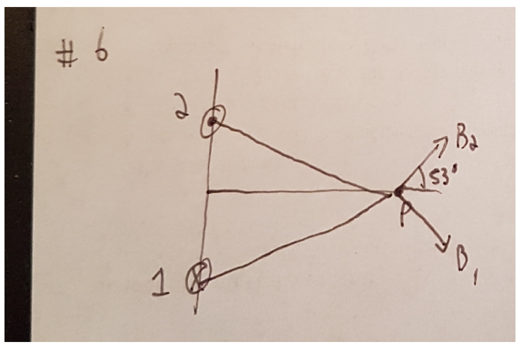
As for the direction, since the $\vec{\mu}$ is pointing in the $(-\hat{\imath}, +\hat{k})$ direction and that the B field is in the $+\hat{k}$ direction, the cross product of the $-\hat{\imath} \times \hat{k}$ is all we care about. Therefore, the torque is $\boxed{\vec{\tau} = 0.072\hat{\jmath}Nm}$.

6. (a) To get the resultant magnetic field, it is important to find the magnitude and direction of the two B field vectors at P. We will need the distances which are both equal at a value of 0.5m. Let us start with the first one:

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \cdot 10^{-7} \cdot 20}{2\pi \cdot 0.5} = 8 \cdot 10^{-6} = B_2 \tag{7}$$



Since the two wires are carrying the same current at the same distance from P, the magnitudes of their magnetic fields will be the same, only their directions will be different. From the diagram (and some trig), we can see that the angle that B_2 makes is 53° and the angle that B_1 makes is −53°. This can be seen from the picture below:



It is therefore obvious that there y components will cancel and that the resulting x component will be double one of them. In conclusion: $\boxed{B_{tot} = 2 \cdot 8 \cdot 10^{-6} cos(53) = 9.6 \cdot 10^{-6} \hat{\imath} T}$

(b) Since we are looking for the force on a wire and we know all the parameters, let us simply plug in the correct values in $F = ILB$ since everything is perpendicular.

$$F = ILB = 30 \cdot 0.1 \cdot 9.6 \cdot 10^{-6} = 2.88 \cdot 10^{-5} N \tag{8}$$

Therefore, the force is $\boxed{2.88 \cdot 10^{-5} \hat{\jmath} N}$

7. To get the force on the electron, the first thing we will need is the magnetic field stemming from the wire. We can obtain that by using

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \cdot 10^{-7} \cdot 1.5}{2\pi \cdot 0.1} = 3 \cdot 10^{-6} T \tag{9}$$

Using the right hand rule, we can figure out that the direction of the B field is into the page which we will call $-\hat{k}$.

Now that we have the magnetic field stemming from the wire, we can calculate the force on the electron.

$$F = qvB = 1.6 \cdot 10^{-19} \cdot 5 \cdot 10^{4} \cdot 3 \cdot 10^{-6} = 2.4 \cdot 10^{-20} N \tag{10}$$

Since we have the magnitude, the only thing remaining is the direction which we can get by using the right hand rule and then flipping the sign of the answer as it is an electron that is moving. $\boxed{\vec{F} = 2.4 \cdot 10^{-20} - \hat{j} N}$

8. To start, do not think of this as a dynamic problem where the wire will move up or down and eventually find the right height, think of it as the wire has been oscillating for awhile and found its equilibrium position. At that equilibrium position, let us figure out how the forces balance. The 2 forces opposing each other will be the gravitational force (downwards) and the magnetic force between 2 wires (upwards).

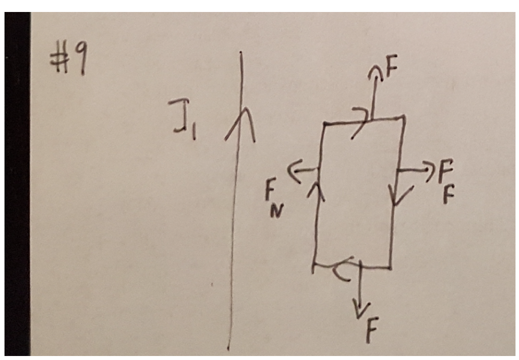
$$F = mg = ILB = IL\frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I^2 L}{2\pi r} \rightarrow r = \frac{\mu_0 I^2 L}{2\pi mg} = 1.02cm \tag{11}$$

Therefore, the wire will settle at a position of $\boxed{1.02cm}$ above the AB wire.

9. We can see from the geometry of this problem that the two segments of wire that are perpendicular to I_1 will cancel as they are the same distance away from I_1 and carry the same current in opposite directions. However, that is not true for the far side and the near side that are parallel to I_1. Let us start by getting the force on the near side:

$$F_N = ILB = IL\frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{4\pi \cdot 10^{-7} \cdot 20 \cdot 10 \cdot 0.2}{2\pi \cdot 0.01} = 8 \cdot 10^{-4} N \tag{12}$$

To help with the directions of all the fores, follow this diagram:



The direction of the force on the near side is, as we can tell from the right hand rule, pointing to the left. Let us now calculate the force on the far side of the loop using the same formula:

$$F_F = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{4\pi \cdot 10^{-7} \cdot 20 \cdot 10 \cdot 0.2}{2\pi \cdot 0.11} = 7.3 \cdot 10^{-5} N \tag{13}$$

The direction of the force on the far side is, as we can tell from the right hand rule, pointing to the right. Let us now calculate the net force on the loop.

The net force on the loop will be $\boxed{F_{net} = +7.3 \cdot 10^{-5} - 8 \cdot 10^{-4} = -7.3 \cdot 10^{-4} N}$ pointed towards the left.

10. Recall that the formula for a coil is $B = \frac{\mu_0 NI}{2r}$. Therefore, we can calculate the required number of turns by using:

$$B = \frac{\mu_0 NI}{2r} \rightarrow N = \frac{2Br}{\mu_0 I} = \frac{2 \cdot 1.26 \cdot 10^{-4} \cdot 0.2}{4\pi \cdot 10^{-7} 2.5} = 16 \tag{14}$$

To achieve the required B field, $\boxed{16 turns}$ need to be present.

11. Like the previous question, let us make sure to use the appropriate formula:

$$B = \frac{\mu_0 NI}{L} = \frac{4\pi \cdot 10^{-7} \cdot 200 \cdot 5}{0.2} = 6.28 \cdot 10^{-3} T \tag{15}$$

Therefore the magnetic field inside the solenoid is $\boxed{6.28 \cdot 10^{-3} T}$

12. (a) Recall that the formula for a slidewire is $\epsilon = Blv$. We can obtain ϵ in this problem by using Ohm's law.

$$v = \frac{\epsilon}{Bl} = \frac{IR}{Bl} = \frac{1.5 \cdot 0.25}{0.5 \cdot 0.5} = 1.5 m/s \tag{16}$$

The bar is moving at $\boxed{1.5 m/s}$

(b) Recall from mechanics (this one may be deeper in your memory) that $P = \vec{F} \cdot \vec{v}$. Using this formula, we can calculate the power expended by pulling the bar

$$P = ILB \cdot v = 1.5 \cdot 0.5 \cdot 0.5 \cdot 1.5 = 0.56 W \tag{17}$$

The power expended by the bar is $\boxed{P = 0.56 W}$

(c) We know from conservation of energy that they should be the same, but let's calculate the Joule heating via $\boxed{P = RI^2 = 0.25 \cdot 1.5^2 = 0.56 W}$.

13. (a) To get the induced EMF, one must properly use the formula of $\epsilon = N\frac{d\phi}{dt}$. We can start by calculating the magnetic flux: $\phi = \vec{B} \cdot \vec{A} = BAcos(theta) = B \cdot 0.15 \cdot cos(60) = 0.075B$. Therefore, putting this in the induced EMF formula yields:

$$\epsilon = 200\frac{d\phi}{dt} = 200 \cdot 0.075\frac{dB}{dt} = 15 \cdot (0.02 \cdot t + 0.05) = 15 \cdot (0.1 + 0.05) = 2.25V \quad (18)$$

Therefore the induced EMF at 5 seconds is $\boxed{2.25V}$

(b) Using simple Ohm's law yields $\boxed{V = IR \rightarrow I = \frac{V}{R} = \frac{2.25}{1.5} = 1.5A}$

(c) As seen from above, the current would go in the clockwise direction.