Supplementary Problems Set 2

1. (a) Strategy: To get the average, let us compute the amount of charge at both moments. By plugging in $t = 2$ into the equation, we obtain $Q(2) = 4 \cdot 2^3 + 5 \cdot 2 + 6 = 48$. By plugging in $t = 4$, we obtain $Q(4) = 4 \cdot 4^3 + 5 \cdot 4 + 6 = 282$. Recall that the current is given by the expression $\boxed{I = \frac{dQ}{dT} \approx \frac{\Delta Q}{\Delta t} = \frac{282 - 48}{2} = 117A}$

   (b) To get the power dissipated at $t = 2$, we need the current. If we re-use the same expression as above, we obtain that $I = \frac{dQ}{dt} = 12 \cdot t^2 + 5$ therefore the current at 2 seconds is 53 A. Since we know that $\boxed{P = RI^2 = 12 \cdot 53^2 = 3.37 \cdot 10^4 W}$

2. (a) This one is simple: $V = IR$ so $\boxed{R = \frac{V}{I} = \frac{15}{0.4} = 37.5\Omega}$

   (b) To get the resistivity ρ, we can use the formula on the formula sheet $R = \frac{\rho L}{A}$ and do some algebra. To obtain ρ, we can figure out that $\boxed{\rho = \frac{RA}{L} = \frac{37.5 \cdot \pi(10^{-3})^2}{3} = 3.9 \cdot 10^{-5}\Omega m}$

   (c) Figure out the power by straight up using the $P = VI$ formula to obtain that $\boxed{P = 15 \cdot 0.4 = 6W}$

   (d) In order to produce a larger current, it would need to produce a larger voltage so we cannot simply use $P = VI$ to solve this equation. Let us use the one thing that does stay the same in this problem, the resistance. Therefore, we should use $P = RI^2$ where P would be 15 and R is 37.5. Performing the algebra yields $\boxed{I = \sqrt{\frac{P}{R}} = \sqrt{\frac{15}{37.5}} = 0.63A}$

3. Even though this question is in two parts, we cannot do one part without doing the other. This will be a system of 2 equations and 2 unknowns. We know that running the current one way gives us a voltage larger than the EMF and running the current the other way will give us a voltage lower than the EMF. We can therefore set-up the 2 equations.

$$10 = \epsilon - rI_1 \tag{1}$$
$$16 = \epsilon + rI_2 \tag{2}$$

   We therefore have 2 equation and 2 unknowns. Solving this system for r, ϵ yields $\boxed{r = 1\Omega}$ and $\boxed{\epsilon = 14V.}$

4. Let us realize right away that we only care about the difference in potential so the 64V and the 16V shouldn't throw you off.

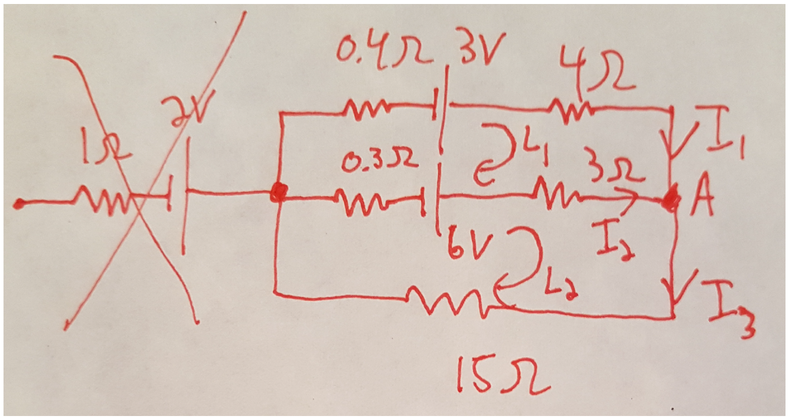
(a) When the switch is open, is is just 2 branches of a parallel circuit each having the entire voltage of 48V. If we are to determine the voltage on the 6Ω resistor, then we only need the current on the top branch. $I = \frac{V}{R} = \frac{48}{18} = 2.7A$. The potential at A will be given by $V_A = 2.7 \cdot 12 = 32V$. Since we started with 64V and lost 32V in the top branch, we are left with 32 V at A. If we use the same procedure for the bottom branch, we get that the current is 3A and that $V_B = 3 \cdot 3 = 9V$. Since we started with 64V and lost 9V in the bottom branch, we are left with 55V at B. The potential difference $\boxed{V_{AB} = 32 - 55 = -23V}$

(b) When the switch is closed, then we have 2 separate parallel circuits. One where the 12 and 3 are in parallel and one where the 6 and the 13 are in parallel. This one is a little more complicated. Let us begin by calculating the equivalent resistance of this new circuit: $(12^{-1} + 3^{-1})^{-1} + (6^{-1} + 13^{-1})^{-1} = 6.5\Omega$. Given this R_{eq}, we can get the total current as $I_{tot} = 7.37A$.

Let us now figure out how much voltage would be lost on each parallel piece. Since the equivalent resistance of the first piece is 2.4Ω and the equivalent resistance of the second piece is 4.1Ω, we can calculate that the voltage lost on each of the two pieces is respectively $V_1 = 7.37 \cdot 2.4 = 17.7$ and $V_2 = 7.37 \cdot 4.1 = 30.3$.

Finally, we have to dig down to the last level to obtain the current in the individual resistors. Figuring all 4 currents in order yields $I_1 = \frac{17.7}{12} = 1.48A$, $I_2 = \frac{17.7}{3} = 5.9A$, $I_3 = \frac{30.3}{6} = 5.05A$, $I_4 = \frac{30.3}{13} = 2.33A$. To answer how much current has gone in the middle branch, let us calculate the difference in the bottom (or top branch). $\boxed{5.9 - 2.33 = 3.57A}$

5. With all the questions asked, it should be clear that we will need to use Kirchoff here.



Starting from the circuit, we can right away remove the left part of the circuit as it does not form a closed loop with anything. Therefore no current will go into or leave the left branch where B is. That doesn't mean that the power source won't play a role later. Let us now set-up our equations according to the following diagram:

$$I_1 + I_2 = I_3 \tag{3}$$
$$-3.3I_2 + 6 - 15I_3 = 0 \tag{4}$$
$$-4.4I_2 + 3 - 6 + 3.3I_2 = 0 \tag{5}$$

Using the loop equations, we can obtain that $I_1 = \frac{-3+3.3I_2}{4.4}$ and that $I_3 = \frac{6-3.3I_2}{15}$. Putting both of these into the junction equation yields $\boxed{I_2 = 0.55A, I_3 = 0.28A, I_1 = -0.27A}$. Where the minus sign indicates that the I_1 current actually runs opposite the direction indicated initially in the diagram (which is why it is irrelevant to spend time deciding current and loop directions in the beginning, it will come out of the math).

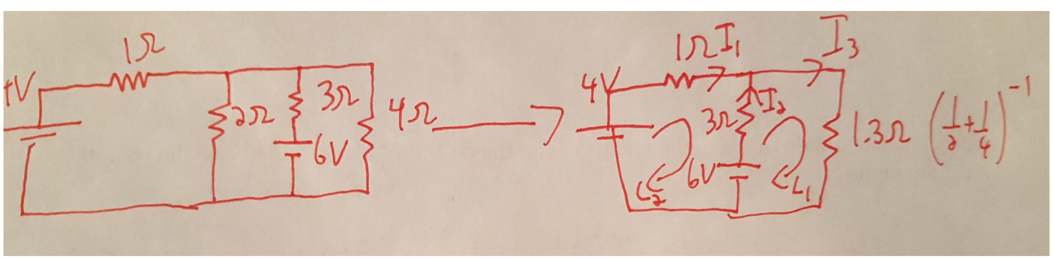
(a) We have just figured out this part.

(b) Since we know that point B is useless in this circuit, we can take any of the three paths from left to right to obtain that $V_A = 15 \cdot I_3 = 4.2V$. However, since we are looking for the potential difference between A and B, the 2V power supply will play a role. If you imagine connecting a multimeter between A and B, you would effectively close the circuit allowing the 2V from the power supply to contribute to the voltage gained/lost in the branches. Therefore $\boxed{V_{AB} = 4.2V + 2 = 6.2V}$

(c) Recall that the current was negative in that branch so it is like we are charging ϵ_1 therefore the terminal voltage will be higher than the EMF. $\boxed{V_{T1} = \epsilon_1 + I_1 \cdot 0.4 = 3.11V}$

(d) It is definitely absorbing energy (since it is charging) so we can just take $P = V_{T1}I_1 = 3.08 \cdot 0.27 = 0.83W$

6. This is another Kirchoff problem that we can set-up by looking at the following diagram and circuit simplification:



$$I_1 + I_2 = I_3 \tag{6}$$
$$4 - I_1 + 3I_2 - 6 = 0 \tag{7}$$
$$6 - 3I_2 - 1.3I_3 = 0 \tag{8}$$

Isolating currents in the loop equations yields: $I_1 = 3I_2 - 2$ and $I_3 = \frac{6-3I_2}{1.3}$. Using these equations and solving for all three currents yields $\boxed{I_1 = 1.15A, I_2 = 1.05A, I_3 = 2.2A}$.

7. (a) This is simply found by looking at the formula on the formula sheet and plugging numbers... $\boxed{C = \frac{k\epsilon_0 A}{d} = \frac{2.3 \cdot 8.85 \cdot 10^{-12} \cdot 0.04}{0.0003} = 2.7 \cdot 10^{-9} F}$

   (b) Plug and chug: $\boxed{U = \frac{CV^2}{2} = \frac{2.7 \cdot 10^{-9} \cdot 100^2}{2} = 1.35 \cdot 10^{-5} J}$

8. (a) To get the equivalent capacitance, remember your rules and calculate: $C_2 + C_3 = 5\mu F$. Then that is in series with C_1 so $\left(C_1^{-1} + C_{23}^{-1}\right)^{-1} = 4\mu F$. Then that is in parallel with C_4 so $\boxed{C_{123} + C_4 = 9\mu F}$.

   (b) Since we already have the charge on C_3, let's use the fact that $V_3 = V_2$ to get the charge on C_2. $V_3 = \frac{Q_3}{C_3} = \frac{9 \cdot 10^{-5}}{3 \cdot 10^{-6}} = 30V = V_2$. So, $Q_2 = C_2 \cdot V_2 = 2 \cdot 10^{-6} \cdot 30 = 60\mu C$. Now, $Q_1 = Q_2 + Q_3 = 150\mu C$. With that, we can figure out that $V_1 = \frac{Q_1}{C_1} = \frac{1.5 \cdot 10^{-4}}{20 \cdot 10^{-6}} = 7.5V$. Therefore, the total voltage on the top branch is 37.5 and the bottom branch must have the same voltage. Finally, $\boxed{Q_4 = C_4 \cdot V_4 = 5 \cdot 10^{-6} \cdot 37.5 = 188\mu C}$.

9. (a) Let us start by making 1 equivalent capacitor. $C_{eq} = \left(9^{-1} + 18^{-1}\right)^{-1} = 6\mu F$. Therfore the time constant $\tau = 6 \cdot 10^{-6} \cdot 2 \cdot 10^6 = 12s$.

   To get the charge on C_1 (which will be the same as the charge on C_2 since they are in series), we can take $\boxed{Q = C_{eq} V \left(1 - e^{\frac{-t}{\tau}}\right) = 6 \cdot 10^{-6} \cdot 30 \left(1 - e^{\frac{-6}{12}}\right) = 7.1 \cdot 10^{-5} C}$

   (b) To get the current use the current equation: $\boxed{I = I_0 e^{\frac{-t}{\tau}} = \frac{30}{2 \cdot 10^6} e^{-0.5} = 9.1\mu A}$

   (c) Let's use the formula that utilizes the numbers we have instead of new numbers: $\boxed{U = \frac{Q^2}{2C} = \frac{(7.1 \cdot 10^{-5})^2}{2 \cdot 18 \cdot 10^{-6}} = 1.4 \cdot 10^{-4} J}$

   (d) Since the circuit has changed, it will have a new time constant $\tau_2 = 6 \cdot 10^{-6} \cdot 5 \cdot 10^6 = 30s$. Since the charge is the same on C_2 or C_1, we can re-use the number we found earlier and use the discharge formula: $\boxed{Q = Q_0 e^{\frac{-t}{\tau_2}} = 7.1 \cdot 10^{-5} e^{\frac{-18}{30}} = 3.9 \cdot 10^{-5} C}$.

   (e) To get the potential difference, we need to start by figuring out the initial voltage on the capacitors. Since we have the charge and the C_{eq}, we can do that no problem