Problem Set #4 – Capacitors

Solutions

PART A: CONCEPTUAL QUESTIONS

(A) If the distance between the plates changes, the capacitance changes.
If *d* increases, *C* decreases, and if the voltage across the capacitor stays the same during the process, the charge on the capacitor decreases.
If *d* decreases, *C* increases and if the voltage across the capacitor stays the same, the charge will increase.

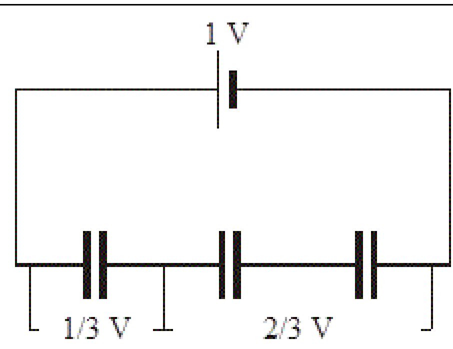
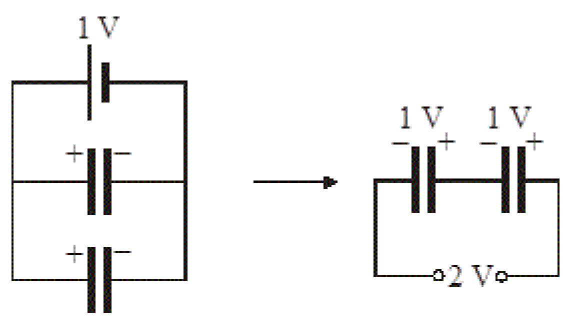
(B) If the capacitor is disconnected from the battery, the charge will stay the same (but as we move the plates, the voltage will change) Moving the plates apart decreases the capacitance, but increases the energy stored in the system. (you should find a formula to check that I'm not kidding you about this) This energy comes from the work done by the external agent in separating the plates against the force of attraction between the plates.

(C) When a dielectric substance is placed in the field between the plates, polarisation of charge takes place in the dielectric. The side of the dielectric nearest the positive plate becomes negative and the side of the dielectric nearest the negative plate becomes positive. The effect of this is similar to bringing the plates closer to each other, and more charge can be placed on the plates with the same battery.

(D) (d)

(E) (c)

(F)

To get 1/3 and 2/3 V:	
To get 2 V:	

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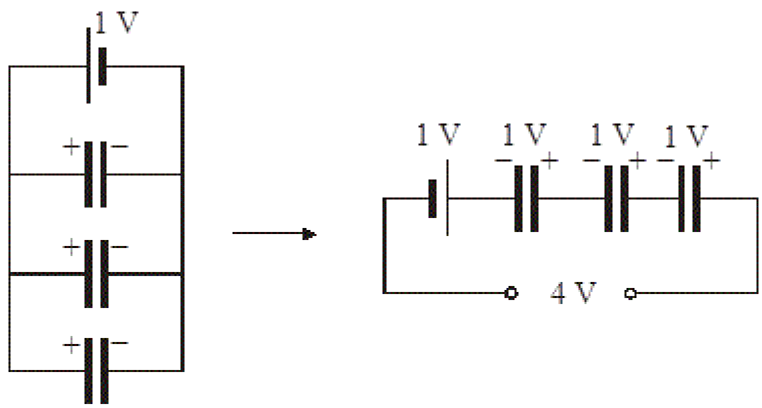
<table>
<tr><td>To get 4 V:</td><td>
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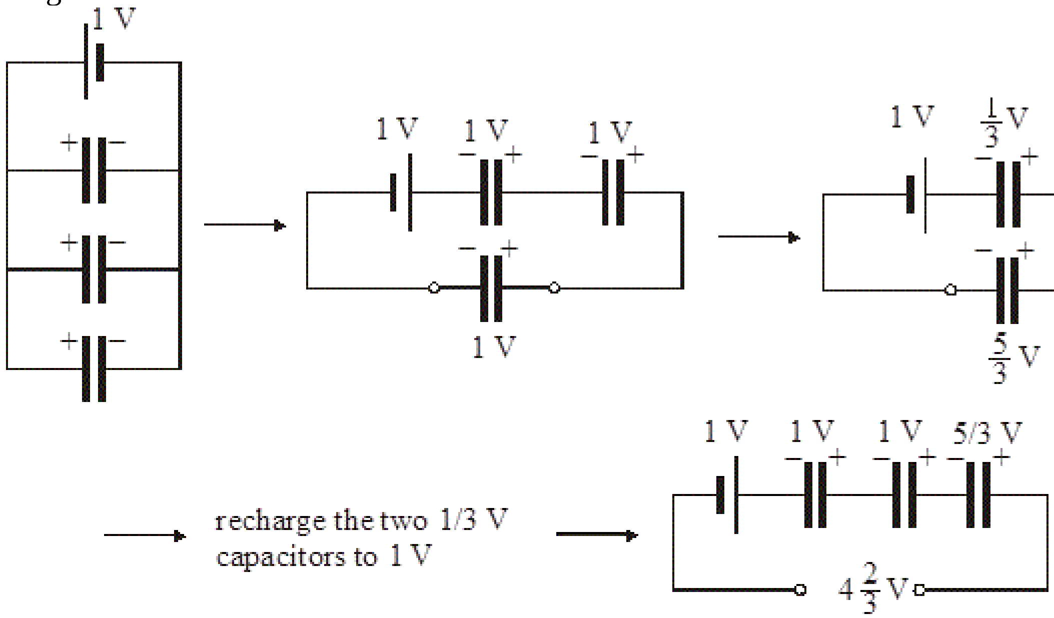
<tr><td colspan="2">To get more than 4 V

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G. The voltage across the capacitor is initially zero, this makes the initial voltage across L2 equal to the emf, therefore L2 is initially very bright, it then dims but it doesn't go out. L1 is initially dim (the initial voltage across it is zero) and it gets gradually brighter. The final brightness of L1 and L2 is the same.

PART B: NUMERICAL QUESTIONS

QUESTION 1

$$Q = CV = 80.0\times10^{-9}\times500.0 = \boxed{4.00\times10^{-5}\ \text{C}}$$

QUESTION 2

(a) The capacitor is disconnected from the charging source so its charge does not change when the dielectric is inserted into it.

$$\Rightarrow Q = V_0 C_0 = VC$$

$$\Rightarrow \frac{V_0}{V} = \frac{C}{C_0} = \frac{200}{50} = 4$$

But $\kappa = \frac{C}{C_0}$ $\quad\Rightarrow \boxed{\kappa = 4}$

(b) The energy stored in the capacitor before the dielectric is inserted is:

$$U_i = \tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2} \times 2.00 \times 10^{-3} \times 200.0^2 = 40.0 \text{ J}$$

The energy stored in the capacitor after the dielectric is inserted is:

$$U_f = \tfrac{1}{2} CV^2 = \tfrac{1}{2} \kappa C_0 V^2 = \tfrac{1}{2} \times 4 \times 2 \times 10^{-3} \times 50^2 = 10.0 \text{ J}$$

Work done $= U_f - U_i = \boxed{-30.0 \text{ J}}$

Since the capacitor has lost energy, it is the electric field in the capacitor that has done the work. The work was done on the dielectric.

QUESTION 3

(a) In order for the electric field across the Pyrex to be its maximum value of 44×10^6 V/m, the space between the plates must be: $d = \frac{6000}{44 \times 10^6} = 1.364 \times 10^{-4}$ m

Since $C = \frac{\kappa \varepsilon_0 A}{d}$

$$A = \frac{Cd}{\kappa \varepsilon_0} = \frac{0.2 \times 10^{-6} \times 1.364 \times 10^{-4}}{5.6 \times 8.85 \times 10^{-12}} = \boxed{0.550 \text{ m}^2}$$

(b) $U = \tfrac{1}{2} CV^2 = \tfrac{1}{2} \times 0.2 \times 10^{-6} \times 6000^2 = \boxed{3.60 \text{ J}}$

QUESTION 4

(a)The equivalent capacitance of the series combination is $C_{eq,S} = \frac{C_2 C_1}{C_2 + C_1} = \frac{2\times 4}{2+4} = \frac{4}{3}\ \mu\text{F}$.

The charge on the combination is $Q_{eq,S} = Q_{2i} = Q_{1i} = C_{eq,S}V = \frac{4}{3}\times 10^{-6}\times 100 = \frac{4}{3}\times 10^{-4} C$.

[The subscript *i* indicates the initial condition and the subscript *f* indicates the final condition]

When the plates are reconnected positive plate to positive plate and negative plate to negative plate, they form a parallel combination in which the total charge is the sum of the individual charges, i.e. $Q_{eq,P} = \frac{4}{3}\times 10^{-4} + \frac{4}{3}\times 10^{-4} = \frac{8}{3}\times 10^{-4} C$

The equivalent capacitance of the parallel combination is $C_{eq,P} = 4.00 + 2.00 = 6.00\ \mu\text{F}$

The potential difference across the parallel combination is

$$V_P = V_{2f} = V_{1f} = \frac{Q_{eq,P}}{C_{eq,P}} = \frac{\frac{8}{3}\times 10^{-4}}{6\times 10^{-6}} = 44.4\ \text{V}$$

The charge on the 2 μF capacitor is therefore $Q_{2f} = V_{2f}C_2 = 44.4\times 2x10^{-6} = 8.90\times 10^{-5} C$

And the charge on the 4 μF capacitor is $Q_{1f} = V_{1f}C_1 = 44.4\times 4\times 10^{-6} = 1.78\times 10^{-4} C$

4. (b) The initial energy stored in C_1 is $\frac{1}{2}\frac{Q_{1i}^2}{C_1} = \frac{\left(1.33\times 10^{-4}\right)^2}{2\times 4\times 10^{-6}} = \boxed{2.22\times 10^{-3}\ \text{J}}$

The initial energy stored in C_2 is $\frac{1}{2}\frac{Q_{2i}^2}{C_2} = \frac{\left(1.33\times 10^{-4}\right)^2}{2\times 2\times 10^{-6}} = \boxed{4.44\times 10^{-3}\ \text{J}}$

The final energy stored in C_1 is $\frac{1}{2}\frac{Q_{1f}^2}{C_1} = \frac{\left(1.78\times 10^{-4}\right)^2}{2\times 4\times 10^{-6}} = \boxed{3.95\times 10^{-3}\ \text{J}}$

The final energy stored in C_1 is $\frac{1}{2}\frac{Q_{2f}^2}{C_2} = \frac{\left(8.9\times 10^{-5}\right)^2}{2\times 2\times 10^{-6}} = \boxed{1.98\times 10^{-3}\ \text{J}}$

QUESTION 5

(a) The equivalent capacitance of the 3.00-μF and 6.00-μF parallel combination is:

$$C_{eq,3,6} = 3 + 6 = 9.00\ \mu\text{F}$$

The equivalent capacitance of the 18.0-μF and 9.00-μF ($C_{eq,3,6}$) series combination is:

$$C_{eq,18,9} = \frac{18 \times 9}{18 + 9} = 6.00\ \mu\text{F}$$

The equivalent capacitance of the 4.00-μF and 12.0-μF series combination is:

$$C_{eq,4,12} = \frac{4 \times 12}{4 + 12} = 3.00\ \mu\text{F}$$

The equivalent capacitance of the $C_{eq,4,12}$ and $C_{eq,18,9}$ parallel combination, which is the equivalent capacitance of the circuit, is:

$$C_{eq} = 6 + 3 = \boxed{9.00\ \mu\text{F}}$$

(b) The 3.00-μF capacitor is part of the parallel combination $C_{eq,3,6}$. Since the voltage across that combination is 8.00 V, the charge on $C_{eq,3,6}$ is:

$$Q_{eq,3,6} = V_{eq,3,6} C_{eq,3,6} = 8.00 \times 9.00 \times 10^{-6} = 7.20 \times 10^{-5}\ \text{C}$$

Since that combination is in series with the 18.0-μF capacitor, the charge on the 18.0-μF capacitor is also 7.20×10^{-5} C. This means that the voltage across the 18.0-μF capacitor is:

$$V_{18} = \frac{7.2 \times 10^{-5}}{18 \times 10^{-6}} = 4.00\ \text{V}$$

$$\Rightarrow V_{ab} = V_{18} + V_{eq,3,6} = 4.00 + 8.00 = 12.0\ \text{V}$$

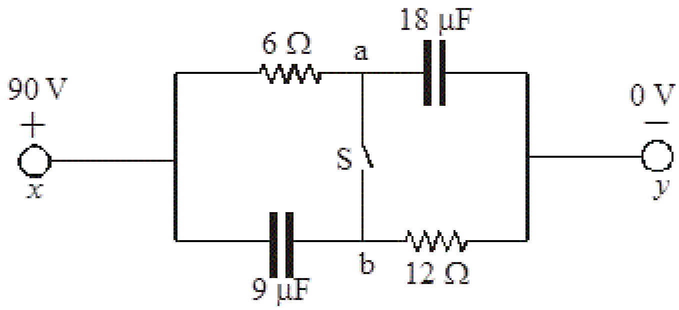
The charge on the 4-μF and 12-μF is the same and is equal to the charge on $C_{eq,4,12}$.

$$\Rightarrow Q_4 = Q_{eq,4,12} = V_{ab} C_{eq,4,12} = 12.0 \times 3 \times 10^{-6} = \boxed{36.0\ \mu\text{C}}$$

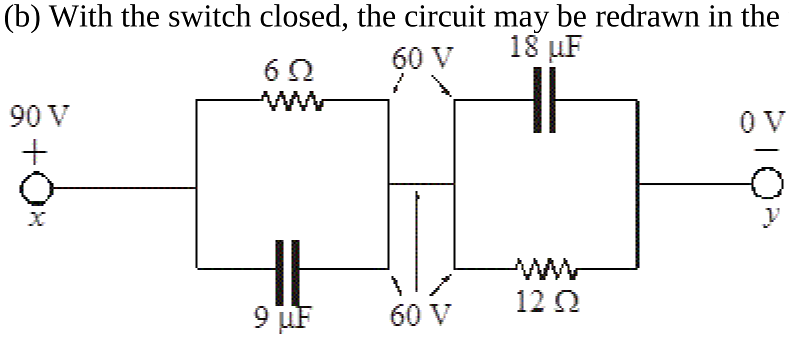
QUESTION 6

(a) With the switch open, there is zero current through the circuit once the capacitors have become charged. Since the current is zero, the voltage drop across the resistors is also zero, i.e.

$$V_a = 90\text{ V},\ V_b = 0\text{ V}$$
$$\Rightarrow V_{xb} = V_x - V_b = 90 - 0 = \boxed{90.0\text{ V}}$$



(b) With the switch closed, the circuit may be redrawn in the following way:



Current flows through the two resistors, which are now connected in series, and of course, no current flows through the capacitors. The voltage drop across the 6.00-Ω resistor is 30.0 V and the voltage drop across the 12.0-Ω resistor is 60.0 V. The potential difference across the 9.00-μF capacitor is $V_9 = 90.0 - 60.0 = \boxed{30.0\text{ V}}$

QUESTION 7

(a) Since the capacitors are in series the charge is the same on both of them and is the same as on the equivalent capacitance that represents them. The equivalent capacitance of the series combination is: $C_{eq} = \frac{4\times 2}{4+2} = \frac{4}{3}\ \mu\text{F}$

The time constant of the charging circuit is: $\tau_C = R_C C_{eq} = 5.00\times 10^6 \times \frac{4}{3}\times 10^{-6} = \frac{20}{3}$ s

$$Q(t) = C_{eq}V\left(1 - e^{-t/\tau_C}\right) = \frac{4}{3}\times 10^{-6}\times 20\times\left(1 - e^{-\frac{10}{20/3}}\right) = \boxed{2.07\times 10^{-5}\text{ C}}$$

The current after 10 s of charge is: $I(t) = \frac{V}{R_C}e^{-t/\tau_C} = \frac{20.0}{5.00\times 10^6}e^{-\frac{10}{20/3}} = \boxed{8.93\times 10^{-7}\text{ A}}$

(b) When the switch is flipped to B, the capacitors discharge through the 6.00-MΩ-3.00-MΩ parallel combination. The equivalent resistance of this combination is $R_D = \frac{3\times 6}{3+6} = 2.00\ \text{M}\Omega$

The time constant of the discharging circuit is: $\tau_D = R_D C_{eq} = 2.00\times 10^{6}\times \frac{4}{3}\times 10^{-6} = \frac{8}{3}$ s

The charge on the capacitors after 10.0 s of discharge is:

$$Q(t) = C_{eq} V e^{-t/\tau_D} = \frac{4}{3}\times 10^{-6}\times 20\times e^{-\frac{10}{8/3}} = \boxed{6.27\times 10^{-7}\ \text{C}}$$

The voltage across the 4-μF capacitor at that time is: $V_4 = \frac{Q(10)}{C_4} = \frac{6.27\times 10^{-7}}{4.00\times 10^{-6}} = \boxed{0.157\ \text{V}}$

The current in the equivalent R after 10 s of discharge is:

$I(t) = \frac{V}{R_D} e^{-t/\tau_D} = \frac{20.0}{2.00\times 10^{6}} e^{-\frac{10}{8/3}} = \boxed{2.35\times 10^{-7}\ \text{A}}$ Therefore, the voltage across the equivalent R is $V = RI = 2.00\times 10^{6}(2.35\times 10^{-7}) = 4.70\times 10^{-1} V$ so the current in the 3MΩ resistor will then be: $I(t) = \frac{V}{R} = \frac{4.70\times 10^{-1}}{3.00\times 10^{6}} = 1.57\times 10^{-7} A$

QUESTION 8

(a) Yes

(b) Circuit 1 $I = \frac{V}{R} = \frac{40.0}{2.00} = 20.0A$

Circuit 2 $I = \frac{V}{R} = \frac{20.0}{8.00} = 2.50A$

(c) In the first circuit the capacitors are connected in series, so the initial voltage across the bulbs is twice the voltage of a single capacitor. The bulbs are connected in parallel and therefore each receives that voltage. The bulbs therefore shine more brightly than in experiment 1. However, the capacitors discharge more quickly because the current in the circuit is greater. In the second circuit the voltage supplied by the capacitors in parallel is the same as in experiment 1 but the bulbs are in series, the current is therefore smaller and so the bulbs shine less brightly. The capacitors discharge less quickly because their combination has more available charge and because the current is smaller.

(d) The bulbs will shine with equal brightness when the current through them is the same. In the first circuit the equivalent capacitance is $C/2 = 0.500$ F, and the equivalent resistance is $R/2 = 2.00\ \Omega$. The time constant of the circuit is $\tau_1 = R_1 C_1 = 2.00\times 0.500 = 1.00$ s.

The total current through the whole first circuit, as a function of time, is given by:
$I_{total}(t) = \frac{V_{01}}{R} e^{-t/\tau_1} = \frac{40}{2} e^{-t/1} = 20e^{-t/1}$.

But the bulbs in the first circuit are in parallel. Since they have equal resistance, they share that current equally, each getting half of that amount:
$I_1(t) = \frac{20}{2} e^{-\frac{t}{1}} = 10e^{-t/1}$.

In the second circuit the equivalent capacitance is $2C = 2.00$ F, and the equivalent resistance is 8.00. The time constant of the circuit is $\tau_2 = R_2 C_2 = 8.00 \times 2.00 = 16.0$ s.
The current as a function of time in this circuit is given by: $I_2(t) = \frac{V_{02}}{R_2} e^{-t/\tau_2} = \frac{20}{8} e^{-t/16}$.

$I_1 = I_2$ when $10e^{-t/1} = \frac{20}{8} e^{-t/16}$

$\Rightarrow 4 = \frac{e^{-t/16}}{e^{-t}} = e^{\frac{15t}{16}}$ $\Rightarrow ln\,4 = \frac{15t}{16}$ $\Rightarrow t = \frac{16\,ln\,4}{15} = \boxed{1.48\text{ s}}$