SOLUTIONS: PROBLEM SET 3

ELECTRIC CURRENT and DIRECT CURRENT CIRCUITS

PART A: CONCEPTUAL QUESTIONS

A. If you connect two in series, you would have $R_{eq} = 200\Omega$.
If you connect two in parallel, you would have $R_{eq} = 50\ \Omega$ $\quad R_{eq} = (\frac{1}{R_1} + \frac{1}{R_2})^{-1}$
Therefore, in order to obtain a 150 Ω resistance, connect the resistors in parallel and in series…
Connecting two in parallel: $R_{eq1} = 50\ \Omega$
Connecting R_{eq1} in series with R: $R_{eq} = 150\ \Omega$.

B. c) The wire has essentially zero resistance, compared to the light bulb, and is in parallel with it. Thus, almost all charges flow through the wire, and practically none through the bulb. (The path of least resistance.)

C. Circuit 1: f) Define three currents (I_1, I_2 and I_3) and do two Kirchhoff's loop rules, and one junction rule, to prove that the currents through each resistor are the same, and the current through the branch is zero.

Circuit 2: f) Because the light bulbs are identical, the potential difference across each is 12 V, and so nothing happens with the switch is closed.

D. Using the circuit:
- a) Dimmer
- b) Increase (*point A should have been at the bottom left corner of the circuit, and point C should have been at the top left corner*)
- c) Decrease
- d) Increase
- e) Increase
- f) Decrease
- g) Nothing
- h) Increase
- i) Increase (point D should have been at the top right corner, while F should have been at the bottom right corner.)

E. The current will be the same because in series, the current in the branch does not change. Even if the light bulbs are different, the current across the resistors will be the same.

F. The current will be different because the light bulbs are in parallel. The current measured going through the battery will be equal to the sum of the currents in each branch. The current will be greater if the resistor in that branch is smaller.

G. In parallel, the current is different but the voltage is the same.
- a) The reading on the voltmeter will be the same since the voltage is the same in parallel branches.
- b) L_2 since the resistance is less and in order to have the same voltage across the branch the current has to be greater.
- c) Note, c) and d) are really the same question. A light bulb which dissipates more energy will be brighter. L_2 will be brighter because it dissipates energy at the rate $P = \frac{V^2}{R}$ and L_2 has less resistance than L_1 (but the same V in parallel).

H. We have to remember the relationship between L, d and R: $R = \rho \frac{L}{A} = \rho \frac{L}{\pi (d/2)^2}$

a) If L is doubled, the resistance R will also double. As long as the diameter is the same. $R \propto L$
b) If d is doubled, the resistance R decreases by a factor of 4. $R \propto d^{-2}$.
c) If L and d are doubled, the resistance R will decrease by a factor of 2.

I. The fundamental principle that allows a battery to deliver charges and energy to a circuit is the difference in reduction and oxidation energies (yes: chemistry in a physics class) between 2 different elements. The key in a battery is to let the ions flow between the anode and cathode, but to prevent the electrons to go directly from the anode to the cathode. This allows the reduction-oxidation (redox) reaction to take place and for electrons to flow between the anode and cathode by going through the connected circuit.

At its heart, this is the technology behind batteries, finding two materials that have the largest possible difference between their reduction and oxidation energies (related to ionization energies, but not the same), putting them together through a path that allows the ions to flow between them, but not the electrons that must take the long way around. Whether, lead-acid (car), Li-ion (phones, lap-tops) or Zn-Mn (remotes, flashlights, normal batteries) all batteries operate using this principle.

No. To explain, let us distinguish between a *discharged* battery, which can be recharged and a *dead* battery meaning that no recharge is possible. The reason behind the death of batteries does not have to do with the lack of charges to move around. From the previous question, we understand the core principle of batteries (a redox reaction), we understand that the creation of the difference in potential that causes electron flow stems from a difference of potential in the redox reaction. Given a sufficient amount of time, the redox reaction of the elements will reach equilibrium and no more ions will flow through the channel. Such an equilibrium will remove the difference in potential between the electrodes and will therefore cause the *death* of the battery since no difference of potential exists between the electrodes. The battery does not run out of charge, but the charges stop flowing.

PART B: NUMERICAL QUESTIONS

QUESTION 1

We obtain the resistance R_A of the conductor:

$$R_A = \rho \frac{L}{\pi r_A^2}$$

with r_A radius of conductor A
r_{out} radius of outer conductor B
r_{in} radius of inner conductor B

Therefore, the transversal area is $\pi(r_{out}^2 - r_{in}^2)$

The ratio between A and B will be (since "rho" and L will be the same, and thus cancel):

$$\frac{R_A}{R_B} = \frac{r_{out}^2 - r_{in}^2}{r_A^2} = \frac{(1.0mm)^2 - (0.50mm)^2}{(0.50mm)^2} = 3.0$$

QUESTION 2

In order to find the carrier charge density needed in the current density equation, we must find the density of atoms.

N: number of atoms
N_A: avogadro's number (6.02×10^{23} mol^{-1})
m: mass
M: element units

$$\frac{N}{N_A} = \frac{m}{M} \rightarrow N = \frac{N_A m}{M}$$

The mass density of substance is $\rho = m \div Volume \rightarrow Volume = m \div \rho$.
The number density of atoms, $n = N \div Volume = N\rho \div m = (\rho N_A) \div M$

With numerical values: $n = \frac{8.9\times10^3 kg/m^3 \cdot 6.02\times10^{23} atoms/mol}{63.5\times10^{-3} kg/mol} = 8.5\times10^{28} atoms/m^3$

The current density is given from $I = qnAv_d$
q: charge
n: number density of charges
A: cross-sectional area
v_d : drift velocity

we solve for v_d: $v_d = \frac{I}{qnA} = \frac{10A}{e \cdot 8.5\times10^{28} atoms/m^3 \cdot 5\times10^{-6} m^2} = 1.5\times10^{-4} m/s$

QUESTION 3

We first need to find the resistance of each resistors from $P = \frac{V^2}{R}$

We have: $R_1 = \frac{120^2}{60} = 240\Omega$ and $R_2 = \frac{120^2}{90} = 160\Omega$

a) when the resistors are connected in series, the current through them is the same and equal to:

$$I = \frac{\varepsilon}{R_1 + R_2} = \frac{120V}{400\Omega} = 0.3A$$

The voltage of each light will be:

$$V_1 = R_1 I = 240\Omega \cdot 0.3A = 72V$$
$$V_2 = R_2 I = 160\Omega \cdot 0.3A = 48V$$
$$V_{tot} = V_1 + V_2 = 72 + 48 = 120V$$

The power dissipated in each resistor is:

$$P_1 = I^2 R_1 = 21.6W$$
$$P_2 = I^2 R_2 = 14.4W$$
$$P_{tot} = I^2 R_{eq} = P_1 + P_2 = 36.0W$$

b) When the resistors are connected in parallel, the voltage across them is the same and equal to:

$$V_{tot} = 120V = V_1 = V_2$$

The equivalent resistance is found from : $R_{eq} = (R_1^{-1} + R_2^{-1})^{-1} = 96\Omega$

The current in the circuit is $I_{tot} = \frac{\varepsilon}{R_{eq}} = \frac{120V}{96\Omega} = 1.25A$

The current in each light bulb is:

$$I_1 = \frac{V}{R_1} = \frac{120V}{240\Omega} = 0.5A$$
$$I_2 = \frac{V}{R_2} = \frac{120V}{160\Omega} = 0.75A$$
$$I_{tot} = I_1 + I_2 = 1.25A$$

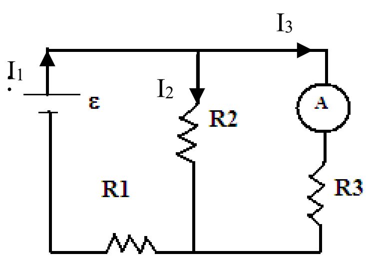
The power dissipated in each resistor is:

$$P_1 = I_1^2 R_1 = 60W$$
$$P_2 = I_2^2 R_2 = 90W$$
$$P_{tot} = I_{tot} R_{eq} = P_1 + P_2 = 150W$$

c) Since the rate of energy dissipated by the circuit is greater when the light bulbs are in parallel ($P_{parallel} > P_{series}$), we conclude that the brightness will also be greater in parallel.

d) The light bulbs in series are more economical because the power dissipated in less. However, there are some inconvenient to this. If one light bulb burns or is unscrewed in series, not more current flows!

QUESTION 4



a) The current in R_1:

$$I_1 = \frac{\varepsilon}{R_1 + (R_2^{-1} + R_3^{-1})^{-1}} = \frac{5.0V}{2.0\Omega + (4.0^{-1} + 6.0^{-1})^{-1}} = 1.14A$$

therefore, the current in R_3 is:

$$I_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - I_1 R_1}{R_3} = \frac{5.0V - (1.14A \cdot 2.0\Omega)}{6.0\Omega} = 0.45A$$

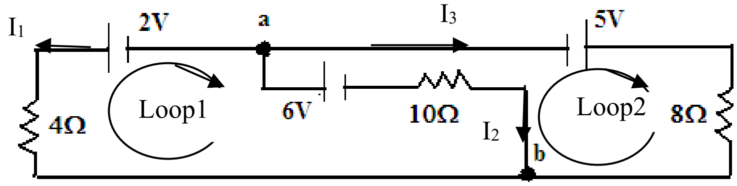
b) We exchange R_1 and R_3 in the above equation:

$$I_3 = \frac{\varepsilon}{R_3 + (R_1^{-1} + R_2^{-1})^{-1}} = \frac{5.0V}{6.0\Omega + (2.0^{-1} + 4.0^{-1})^{-1}} = 0.68A$$

and

$$I_1 = \frac{\varepsilon - V_3}{R_1} = \frac{\varepsilon - I_3 R_3}{R_1} = \frac{5.0V - (0.68A \cdot 6.0\Omega)}{2.0\Omega} = 0.45A$$

QUESTION 5



We have three unknowns (I_1, I_2 and I_3) so three equations are needed.

a) With Kichhoff's law of current and from the above currents in the circuit:

$$(1)\quad I_1 + I_2 + I_3 = 0$$

But, the direction of one current is incorrect. The solution is correct; one current will be negative – thus the math works out for you. Let's apply the voltage law on loop 1:

$$(2)\qquad \Sigma V = -2.0V - 6.0V - 10.0\Omega \cdot I_2 + 4.0\Omega \cdot I_1 = 0$$

The emf are negative since we encounter the + side first. From (2):

$$(3)\qquad I_2 = \frac{-8.0V + 4.0\Omega \cdot I_1}{10.0\Omega} = -0.8A + 0.4I_1$$

from loop 2

$$(4)\qquad \Sigma V = 6.0V - 8.0\Omega \cdot I_3 + 10.0\Omega \cdot I_2 + 5.00 = 0$$

we substitute I_3 from equation (1): $I_3 = -I_1 - I_2$ in equation (4) and solve for I_2:

$$(5)\qquad 11V - 8.0\Omega \cdot (-I_1 - I_2) + 10.0\Omega \cdot I_2 = 0$$

$$(6)\qquad I_2 = \frac{-11.0V}{18.0\Omega} - \frac{8.0\Omega}{18.0\Omega} I_1 = -0.611A - 0.444 \cdot I_1$$

Since equation (3) equals equation (6)

$$-0.8A + 0.4 \cdot I_1 = -0.611A - 0.444 \cdot I_1$$

$$I_1 = 0.224A$$

Therefore we can substitute in equation (6) and (1)

$I_2 = -0.8A + 0.4 \cdot 0.224A = -0.710A$ in the figure, I_2 has to be on the other direction

$I_3 = -I_1 - I_2 = -0.224A - (-0.710A) = 0.486A$

Verify the results:

From the external loop:
$\Sigma V = -2.0V + 5.0V - 8.0\Omega \cdot I_3 + 4.0\Omega \cdot I_1 = 0$

$3V - 8.0\Omega \cdot 0.486A + 4.0\Omega \cdot 0.224A = 0$

b) The potential difference between points a and b is:

$$V_b - 10.0\Omega \cdot I_2 + 6.0V = V_a$$
$$V_b - V_a = 10.0\Omega \cdot 0.710 - 6.0V = 1.1V$$

QUESTION 6

a) The current when the resistors are in series will be:

$$I = \frac{\varepsilon}{R_1 + R_2 + R_3} = \frac{10.0V}{100\Omega + 220\Omega + 680\Omega} = 10mA$$

The current will be the same in all resistors in series. However, the voltmeter will have a different reading, depending on the resistance.

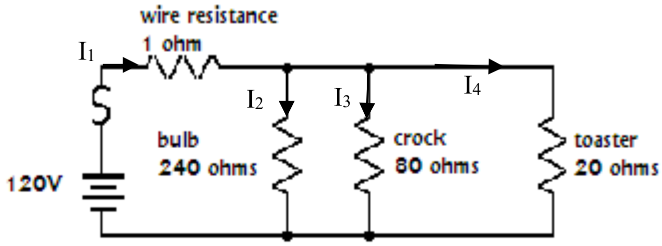
$$V_1 = IR_1 = 1.0V$$
$$V_2 = IR_{23} = 9.0V$$
$$V_3 = IR_3 = 6.8V$$

b) Voltmeter 2, is connected across resistors 2 and 3.

Since $V_2 = I(R_2 + R_3)$ and $I = \frac{\varepsilon}{R_1 + R_2 + R_3}$

$$V_2 = \frac{\varepsilon}{R_1 + R_2 + R_3} \cdot (R_2 + R_3) = \frac{\varepsilon(R_2 + R_3)}{(R_1 + R_2 + R_3)}$$

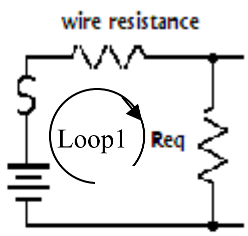
QUESTION 7



The light bulb, the toaster and the crock are in parallel; we can find the equivalent resistance:

$$R_{eq} = (R_{bulb}^{-1} + R_{crock}^{-1} + R_{toaster}^{-1})^{-1} = (240^{-1} + 80^{-1} + 20^{-1})^{-1} = 15\Omega$$

Let consider the left hand side of the circuit has loop 1.



We have a single loop:

$$\Sigma V = 120V - R_{wire} I_1 - R_{eq} I_1 = 120V - 1\Omega \cdot I_1 - 15\Omega \cdot I_1 = 0$$

$$I_1 = 7.5A$$

The current law: $I_2 + I_3 + I_4 = I_1$ and because the branches are in parallel, the voltage across each resistor is the same.

$$V_2 = V_3 = V_4 = V_{eq} = 15\Omega \cdot I_1 = 112.5V$$

$$I_2 = \frac{112.5V}{240\Omega} = 0.46A$$

$$I_3 = \frac{112.5V}{80\Omega} = 1.4A$$

$$I_4 = \frac{112.5V}{20\Omega} = 5.6A$$

Verify the results:

$$I_1 = I_2 + I_3 + I_4 = 7.5A = 0.46A + 1.4A + 5.6A = 7.5A$$

QUESTION 8

a) the ammeter is correctly connected and will read $\varepsilon - I(R+R) = 0 \rightarrow I = 1A$

b) the voltmeter is incorrectly connected (it should be connected in parallel). Since it has an infinite resistance, the voltage read will be $V = \varepsilon = 20V$ (The current will be zero)

c) the ammeter is incorrectly connected (it should be connected in series). Since it has an infinitely small resistance, then it will measure $\varepsilon - IR = 0 \rightarrow I = 2A$ (There will be no current in one of the resistors)

d) the voltmeter is correctly connected and will read: $\varepsilon - I(R+R) = 0 \rightarrow I = 1A \quad V_R = IR = 10V$

QUESTION 9

From the first circuit, the resistance of the non-ideal ammeter can be found:

$$\varepsilon - I(2R + r) = 0$$

$$r = \frac{\varepsilon - 2IR}{I} = \frac{6.000 - 2 \cdot 15 \cdot 196.3 \times 10^{-3}}{196.3 \times 10^{-3}} = 0.565\Omega$$

In the second circuit, the ammeters can be substituted as resistances r = 0.565 ohm.

$$\varepsilon - I(2r + 2R) = 0$$

$$I = \frac{\varepsilon}{2r + 2R} = \frac{6.000}{2 \cdot 0.565 + 2 \cdot 15} = 192.7mA$$

(if both ammeters were ideal, the reading would be 200 mA)

In the third circuit, the equivalent resistance will be:

$$R_{eq} = r + R + \left(r^{-1} + R^{-1}\right)^{-1} = 16.1\Omega$$

The total current is (and that is the current measured in the first ammeter)

$$\varepsilon - IR_{eq} = 0$$

$$I = \frac{\varepsilon}{R_{eq}} = \frac{6.000}{16.11} = 372.4mA$$

The current flowing in the 15-ohm resistor is

$$V_{//} = IR_{//} = 0.372 \cdot 0.544\Omega = 0.2026V$$

$$V_R = V_r$$

$$I_R = \frac{V_R}{R} = \frac{0.2026}{15} = 0.0135A$$

$$I_r = \frac{V_r}{r} = \frac{0.2026}{0.565} = 0.358A$$

QUESTION 10

From the first circuit, the resistance of the non-ideal voltmeter can be found:

$$V_r = V_{150k} = 2.831\ V$$

Using the loop rule (since the voltmeter is non-ideal i.e. non-infinite internal resistance, then the current in the two 150 k-ohm resistors isn't the same):

$$6V - I_1R - I_2R = 0 \rightarrow 6.000 - I_1R - 2.831V = 0 \rightarrow I_1 = \frac{6.000 - 2.831}{150 \times 10^3} = 2.11 \times 10^{-5}A$$

$$2.831V = I_1R_{150\_r} \rightarrow R_{150-r} = \frac{2.831V}{2.11 \times 10^{-5}A} = 134\ k\Omega$$

$$R_{150\_r} = (R^{-1} + r^{-1})^{-1} \rightarrow r = \left(R_{r\_150}^{-1} - R^{-1}\right)^{-1} = 1256\ k\Omega$$

For the second circuit:

$$R_{eq} = 2 \cdot \left[R^{-1} + r^{-1}\right]^{-1} = 2 \cdot 133.9k\Omega = 267.86k\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6.000}{267.86 \times 10^3} = 2.240 \times 10^{-5} A$$

$$V_{//} = R_{//} \cdot I = 133.9 \cdot 2.240 \times 10^{-5} = 2.999V$$

For the third circuit:

$$R_{eq} = R + r + \left[r^{-1} + R^{-1}\right]^{-1} = 1.54E6\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6.000}{1.54E6\Omega} = 3.89 \times 10^{-6} A$$

$$V_1 = rI = 1250k\Omega \cdot 3.89 \times 10^{-6} = 4.87V$$

$$V_{//} = \left[r^{-1} + R^{-1}\right]^{-1} \cdot I = \left[1250^{-1} + 150^{-1}\right]^{-1} k\Omega \cdot 3.89 \times 10^{-6} = 0.522V$$

QUESTION 11

a) before the switch is closed, resistors R_1 and R_2 are in series and R_3 and R_4 are also in series. The equivalent resistance for each combination is:

$$R_{12} = R_1 + R_2 = 4.00\Omega + 10.0\Omega = 14.0\Omega$$
$$R_{34} = R_3 + R_4 = 12.0\Omega + 10.0\Omega = 22.0\Omega$$

R_{12} and R_{34} are in parallel, therefore the equivalent resistance of the system is:

$$(R_{34}^{-1} + R_{12}^{-1})^{-1} = R_{eq} = 8.55\Omega$$

From Ohm's law: $V = RI \rightarrow I = \frac{V}{R_{eq}} = \frac{25.0V}{8.55\Omega} = 2.92A$

The current in each branch will be:

$$I_{12} = \frac{V_{12}}{R_{12}} = \frac{25.0V}{14.0\Omega} = 1.78A$$

$$I_{34} = \frac{V_{34}}{R_{34}} = \frac{25.0V}{22.0\Omega} = 1.14A$$

(and $I_1 = I_2$ and $I_3 = I_4$)

Check the result:

$$I_{tot} = I_{12} + I_{34} = 2.92A$$

b) The switch is now closed. We have to find the equivalent resistance of the circuit where R_1 and R_3 are in parallel and R_2 and R_4 are in parallel.

$$R_{13} = (R_1^{-1} + R_3^{-1})^{-1} = (4.00^{-1} + 12.0^{-1})^{-1} = 3.00\Omega$$

$$R_{24} = (R_2^{-1} + R_4^{-1})^{-1} = (10.0^{-1} + 10.0^{-1})^{-1} = 5.00\Omega$$

R_{13} and R_{24} are in series. The equivalent resistance of the circuit is

$$R_{eq} = R_{13} + R_{24} = 3.00 + 5.00 = 8.00\Omega$$

From Ohm's law: $V = RI \rightarrow I = \frac{V}{R_{eq}} = \frac{25.0V}{8.00\Omega} = 3.13A$

The current through each resistor; we have to find the voltage across each resistor, and the voltage across R_1 and R_3 is the same, the voltage across R_2 and $R_4$is the same.

$$I_1 = \frac{V_{13}}{R_1} = \frac{IR_{13}}{R_1} = \frac{3.13A \cdot 3.00\Omega}{4.00\Omega} = 2.34A$$

$$I_2 = \frac{V_{24}}{R_2} = \frac{IR_{24}}{R_2} = \frac{3.13A \cdot 5.00\Omega}{10.0\Omega} = 1.56A$$

$$I_3 = \frac{V_{13}}{R_3} = \frac{IR_{13}}{R_3} = \frac{3.13A \cdot 3.00\Omega}{12.0\Omega} = 0.78A$$

$$I_4 = \frac{V_{24}}{R_4} = \frac{IR_{24}}{R_4} = \frac{3.13A \cdot 5.00\Omega}{10.0\Omega} = 1.56A$$

Check the result:

$$I = I_1 + I_3 = I_2 + I_4$$
$$3.13A = 2.34A + 0.78A = 1.56A + 1.56A$$
$$3.13 \approx 3.12A \approx 3.12A$$

c) Before the switch is closed: R_1 and R_2 are in series and R_3 and R_4 are in series. From the results in part a), we can find the voltage across each resistor.

$$V_1 = R_1 I_1 = I_{12} R_1 = 1.78A \cdot 4.00\Omega = 7.14V$$
$$V_2 = R_2 I_2 = I_{12} R_2 = 1.78A \cdot 10.0\Omega = 17.86V$$
$$V_3 = R_3 I_3 = I_{34} R_3 = 1.14A \cdot 12.0\Omega = 13.64V$$
$$V_4 = R_4 I_4 = I_{34} R_4 = 1.14A \cdot 10.0\Omega = 11.36V$$

Check the result:

$$V_{tot} = V_1 + V_2 = V_3 + V_4$$
$$25.0V = 7.14V + 17.86V = 13.64V + 11.36V$$
$$25.0V = 25.0V = 25.0V$$

The voltage at points a and b:

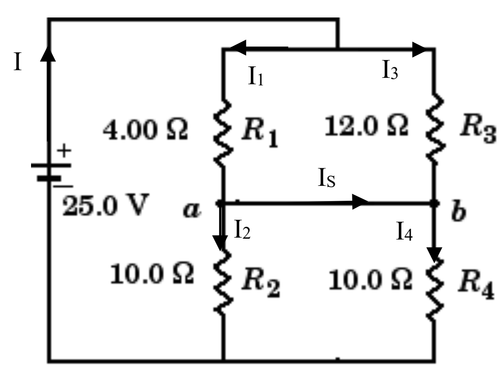
$$V_a = 25.0V - V_1 = 25.0V - 7.14V = 17.9V$$
$$V_b = 25.0V - V_3 = 25.0V - 13.64V = 11.4V$$

The potential difference between point a and b:

$$V_{ab} = |V_b - V_a| = |11.36V - 17.85V| = 6.49V$$

After the switch is closed, $V_{ab} = 0$ because connecting points a and b forces the potential at points and b to be the same, so that their potential difference is zero.

d)



I_S is the current through the switch

$$I_1 = I_S + I_2$$
$$I_S = I_1 - I_2 = 2.34A - 1.56A = 0.78A$$

As a double check (!!) we apply Kirchhoff's current rule at point b

$$I_S + I_3 = I_4$$
$$I_S = I_4 - I_3 = 1.56A - 0.78A = 0.78A$$

The current is flowing from a to b (because the current found is positive)

QUESTION 12

The total power when the if the electric frying pan, the coffee and the toaster are used:

$$P_{tot} = P_{pan} + P_{coffee} + P_{toaster} = 1000W + 600W + 700W = 2300W$$

The current flowing in the circuit is:

$$P = VI \rightarrow I = P \div V = 2300 \div 120 = 19.1A$$

$I_{system} \langle I_{fuse}$ therefore, Jim should make the toasts (Jenny will be happy!)

However, if the overhead light is on, the total power will increase;

$$P_{tot} = P_{pan} + P_{coffee} + P_{toaster} + P_{light} = 1000W + 600W + 700W + 100W = 2400W$$

The current flowing in the circuit is:

$$P = VI \rightarrow I = P \div V = 2400 \div 120 = 20A$$

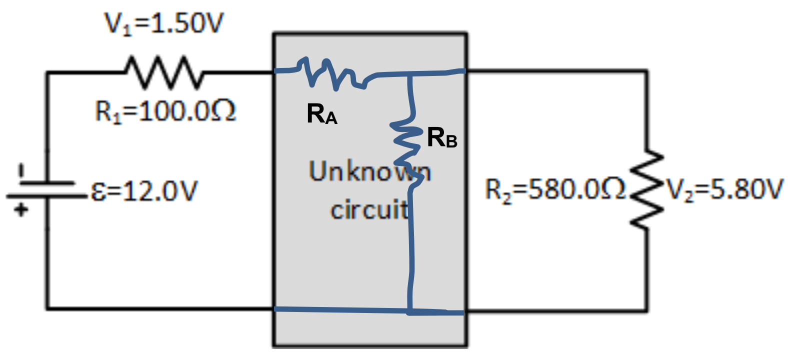
$I_{system} = I_{fuse}$ therefore, the overhead light should not be on.

QUESTION 13

The current through the 100 Ω resistor is: I = 1.5 / 100 = 15 mA, while the current in the 580 Ω resistor is: I = 5.8 / 580 = 10 mA. Since the two currents are different, these two resistors cannot be in series (thus simply connecting the top two wires together with a resistor and the bottom two wires together with a resistor will not work).

One thing to notice is the current on the left side of the box is five milliamps greater than the current on the left. Thus, the current will be broken up inside the box.

Consider adding the following two resistors:



The current through R_A must be 15 mA. The current through R_B must be 5 mA. Using the loop rule, we know that the voltage across R_B must be 5.80V (it is in parallel with the 580 Ω resistor). Thus, its resistance is: $R_B = 5.8 / 0.005 = 1160\ \Omega$.

The voltage drop across R_A must be: $12 - 5.8 - 1.5 = 4.7$ V (so that the loop rule works)

Thus, resistance R_A is: $R_A = 4.7 / 0.015 = 313\ \Omega$