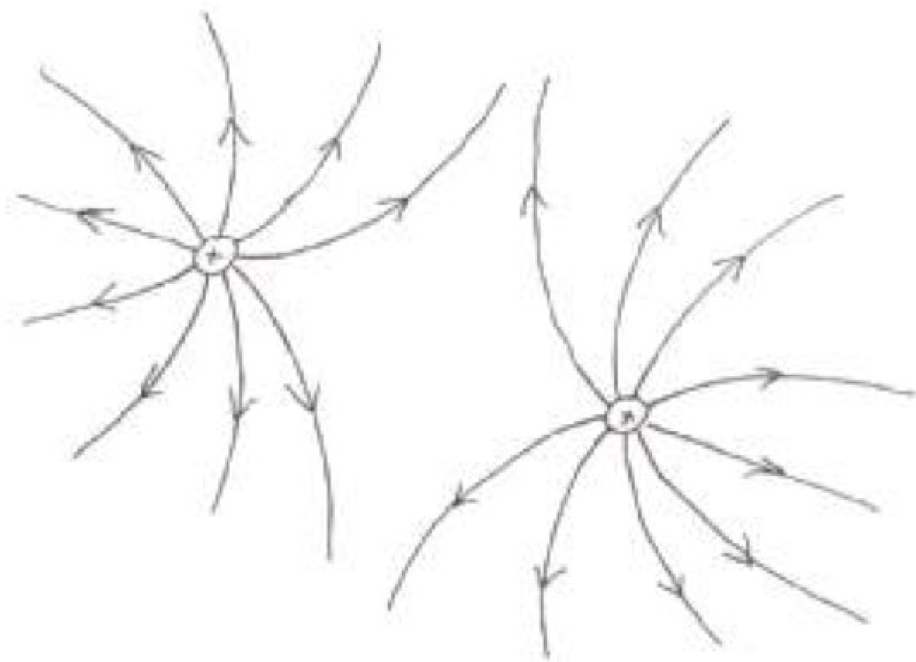
SOLUTIONS: PROBLEM SET 1
ELECTRIC FORCE AND ELECTRIC FIELD

CONCEPTUAL QUESTIONS

A.



B. Charges in the uncharged sphere respond to the electric field produced by the charged sphere. The two *halves* of the uncharged sphere become equally and oppositely charged, with the side close to the charged sphere having the opposite charge to the charged sphere. The side closer to the charged sphere gets attracted and the side further gets repelled, but the attractive force is greater ($F \propto 1/r^2$, r smaller for the closer side). Therefore, the **net force is attractive**.

The instant the sphere touches, charges flow from one sphere into the other (they are metal and conduct charges easily), making them both charged similarly. Like charges repel, therefore they get pushed apart.

We know that afterward both spheres are charged with the same charge, but we have no way of knowing if they are positive or negative. If the charged sphere had been positive, then electrons would have flowed into it from the uncharged sphere, making it less positive and the other sphere positive. If the charged sphere had initially been negative, electrons would have flowed from it into the uncharged sphere, making it less negative and the other sphere negative as well.

C. a) Different clothing fabrics have different affinities for electric charges. When rubbed together, some clothing items tend to *grab* electrons from others, making some negatively charged and some positively charged. As a result, the oppositely charged items attract each other. By the way, since charged objects will also be attracted to neutral objects (see question C), charged clothing will also be attracted to human legs, arms, etc. (Oh, the heartbreak of static cling!)

b) The effect would be much smaller, since there would be less tendency for charges to be transferred if all the material had the same affinity for electrons. (Try this at home!)

c) For the same reason discussed in a), bits of clothing become charged as they rub together. If the charge build-up is sufficient, there might be a spark as the electrons *jump* back across space. (This is referred to as *dielectric breakdown*, when the electric field magnitude becomes stronger than the *dielectric strength* of the medium between the charges. In this case that medium is air; dry air has a dielectric strength of $3\text{x}10^6$ N/C. Descriptively, what happens is this: the electric field is so strong that electrons are pulled free from the negatively charged piece of material. These electrons crash into molecules of air, ionizing them and liberating even more electrons. Essentially, we end up with an avalanche of charge, and the air conducts electrons from the negative to the positive material. This happens with amazing quickness. The air is suddenly heated and expands, causing the sound. As the electrons recombine with the ionized atoms in the air, light is emitted. If this description sounds a lot like thunder and lightning, that's essentially what it is, on a tiny scale.)

d) Water molecules are polar. Warm, humid air contains a large number of these polar molecules. The polar molecules tend to *steal* excess charge from a charged object, making it difficult for any material to accumulate a significant amount of excess charge.

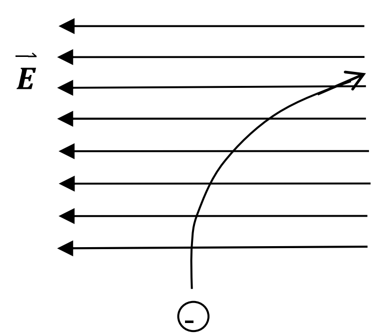
D. a) **positively charged**

E. b) $\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$ (Newton's 3rd law!!)

F. d) **right (+$\hat{\imath}$)**

G. c) **accelerate to the right**

H.



I.

- The negative end of the dipole will be pulled toward it charge *Q*, while the positive is pushed away. At that instant, the negative end is closer to *Q* than the positive end. Thus, *Q* will <u>attract the negative end just be little more than it will repel the positive end. The net force on the dipole due to *Q* is *attractive*.</u>

- Newton's third law! It will be attracted a little bit more by the negative end because it is closer to that end. And it will be repel by the positive end. The net force acting on *Q* is equal and opposite to the net force acting on the dipole.

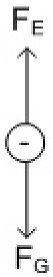
- If *Q* were negative, then it would be attracted to the positive end of the dipole and pulled toward it and the negative end would repel. The force would have the same magnitude as before, but will be in the opposite direction.

PROBLEMS

QUESTION 1

a) $q = -3e = -4.8 \times 10^{-19}\ C.$

Since the charge is at equilibrium, the electric force and gravitational force are equal and opposite.



$$F = F_E = F_G = mg = 3.92 \times 10^{-13}\ N$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{3.92 \times 10^{-13}\ N}{-4.8 \times 10^{-19}\ C} = \boxed{-8.17 \times 10^{5}\ \hat{\jmath}\ N/C}$$

b) $\boxed{-9.81\ \hat{\jmath}\ N/kg}$ (the vector is pointing downward)

QUESTION 2

a) The magnitude of the electric force is given by:

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_o}\frac{|q_1 q_2|}{r^2} = \left|\frac{k_e q_1 q_2}{r^2}\right|$$

$$= \frac{1}{4\pi(8.85\times10^{12}\frac{C^2}{N\cdot m^2})}\left|\frac{(1.602\times10^{-19}C)^2}{(10^{-10})^2}\right| = \boxed{2.3 \times 10^{-8}\ N}$$

b) the magnitude of the gravitational force is given by:

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_G\right| = \frac{|Gm_1 m_2|}{r^2}$$

$$= \left|\frac{\left(6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}\right)(9.1\times10^{-31}kg)(1.6710^{-27}kg)}{(10^{-10})^2}\right| = \boxed{1.0 \times 10^{-47}\ N}$$

Note that $F_E >> F_G$

c) Halving the distance r increases the force $\left|\vec{F}\right|$ by a factor of 4.
Doubling one charge q, doubles the electric force

Therefore, the new electric force: $F_{new} = 8F_{E-original} = 1.8 \times 10^{-7}\ N$

Doubling one mass (i.e 2 protons) doubles the gravitational force. $F_{new} = 8F_{G-original} = 8 \times 10^{-47}\ N$

A helium nucleus (2 protons and 2 neutrons) has 4 times the original mass. $F_{new} = 16F_{G-original} = 1.6 \times 10^{-46}\ N$

QUESTION 3

a) We have two unknowns. We need two equations! If $q_1 = q$ then $q_2 = Q_{tot} - q = 5 \times 10^{-5} - q$

We can isolate the unknown charge q in the force equation.

$$|\vec{F}_{12}| = \frac{1}{4\pi\varepsilon_o}\frac{|q_1 q_2|}{r^2} = \left|\frac{k_e q_1 q_2}{r^2}\right|$$

$$= \left|\frac{\left(8.99\times10^9\frac{N\cdot m^2}{C^2}\right)q(5\times10^{-5}-q)}{r^2}\right|$$

$$F \cdot r^2 = 5 \times 10^{-5} k_e \cdot q - k_e \cdot q^2$$
$$\rightarrow \left(8.99 \times 10^9 \frac{N\cdot m^2}{C^2}\right) q^2 - 4.5 \times 10^5 q + 4 = 0$$

$$q = \frac{4.5 \times 10^5 \pm \sqrt{(4.5 \times 10^5)^2 - 4\left(8.99 \times 10^9 \frac{N\cdot m^2}{C^2}\right)(4)}}{1.8 \times 10^{10}}$$

$$\boxed{q = 3.85 \times 10^{-5}\ C} \quad \text{or} \quad \boxed{q = 1.15 \times 10^{-5}\ C}$$

$q_1 = q$ is either these; $q_2 = 5 \times 10^{-5} - q$ is the other

Parts b) and c)

<u>For the sphere with $3.85 \times 10^{-5}\ C$:</u>

Number of electrons removed: $\frac{3.85\times10^{-5}\ C}{1.602\times10^{-19} C/e} = 2.47 \times 10^{14} e$

Mass of these electrons: $2.47 \times 10^{14}(9.11 \times 10^{-31} kg) = 2.25 \times 10^{-16} kg$

% of mass: $\frac{2.25\times10^{-16} kg}{3\times10^{-3}} \cdot 100 = 7.50 \times 10^{-12}\ \%$

<u>For the sphere with $1.15 \times 10^{-5}\ C$</u>

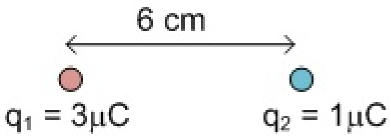
Number of electrons removed: $\frac{1.15\times10^{-5}\ C}{1.602\times10^{-19} C/e} = 7.19 \times 10^{13} e$

Mass of these electrons: $7.19 \times 10^{13}(9.11 \times 10^{-31} kg) = 6.55 \times 10^{-17} kg$

% of mass: $\frac{6.55\times10^{-17} kg}{3\times10^{-3}} \cdot 100 = 2.18 \times 10^{-12}\ \%$

QUESTION 4

Let set $q_1 = 3\mu C$ and $q_2 = 1\mu C$



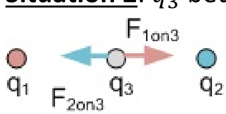
For the net force to be zero, $q_3(6\mu C)$ must be placed on line passing through q_1 and q_2.

$\vec{F}_{net} = \vec{F}_{13} + \vec{F}_{23} = 0 \quad \rightarrow \vec{F}_{13} = -\vec{F}_{23}$ (equal and opposite since the net force = 0)

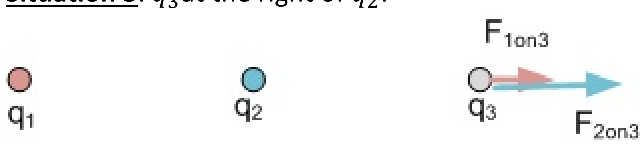
Three situations to consider:

- **Situation 1**: q_3 at the left of q_1:

- **Situation 2**: q_3 between q_1 and q_2:



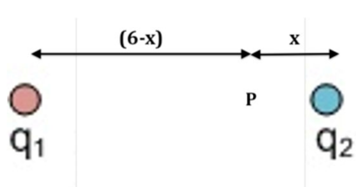
- **Situation 3**: $q_3$at the right of q_2:



In situations 1 and 3, the net force is not equal to zero. So q_3 must be placed somewhere between q_1 and q_2.

Note that this would not be the case if q_1 and q_2 were oppositely charged **(try it!).**

According to situation 2 the force (and so the field!) is zero at distance x from the smallest charge:



$$|\vec{F}_{13}| = |\vec{F}_{23}|$$

$$\left|\frac{k_e q_1 q_3}{r_{13}^2}\right| = \left|\frac{k_e q_2 q_3}{r_{23}^2}\right| \rightarrow \left|\frac{k_e q_1 q_3}{(6-x)^2}\right| = \left|\frac{k_e q_2 q_3}{x^2}\right|$$

$$q_1 x^2 = q_2(6-x)^2$$

$$(q_1 - q_2)x^2 + 12q_2 x - 36q_2 = 0$$

$$2 \times 10^{-6}x^2 + 12 \times 10^{-6}x - 36 \times 10^{-6} = 0$$

Using the quadratic equation (again… it might be a good idea for you to remember it!), we get:

$$x = 3 \pm 3\sqrt{3}$$

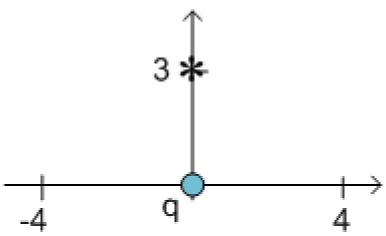
$$x = 2.20\ cm \quad \text{or} \quad x = -19.2\ cm$$

We'll choose the $x = 2.20\ cm$ because the other one ($x = -19.2\ cm$) puts q_3 to the right of q_2. Which of course makes no sense! The reason it came about is that the equation doesn't consider the fact that $\vec{F}_{23}$ changes direction as x gets bigger than 6 cm.

So the charge q_3 must be placed between the other charges, 2.20 cm to the left of q_2.

Note: This problem could have been solved equivalently by finding the point P at which the electric field due to q_1 and q_2 is zero ($\vec{E}_{1P} + \vec{E}_{2P} = 0$). **Why?? Try it!**

QUESTION 5



The electric field of a point charge is given by: $\vec{E}(P) = k_e \frac{q}{r^2}\hat{r}$. The direction of the E-field produced by a positive point charge is *away from the charge*.

a) For the point located at -0.03 m on the x-axis:

$$\vec{E}(x = -0.03m) = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{4 \times 10^{-6}}{0.03^2}(-\hat{\imath}) = -4.00 \times 10^7 \hat{\imath}\ N/C$$

b) the point located at 0.03 m on the y-axis:

$$\vec{E}(y = 0.03m) = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{4 \times 10^{-6}}{0.03^2}(\hat{\jmath}) = 4.00 \times 10^7 \hat{\jmath}\ N/C$$

c) for the point located at (0.04; 0.03)m:

the distance at which this point is located from the charge: $|\vec{r}| = \sqrt{0.03^2 + 0.04^2} = 0.05\ m$

The difference between the point P and the charge is $\vec{r} = (0.04\hat{\imath} + 0.03\hat{\jmath})\ m$
Using unit vector notation:

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{(0.04\hat{\imath} + 0.03\hat{\jmath})\ m}{0.05\ m} = (0.8\hat{\imath} + 0.6\hat{\jmath})$$

or find the angle $\theta = \tan^{-1}\left(\frac{0.03}{0.04}\right) = 36.8°$

$$\vec{E}(P) = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{|4 \times 10^{-6}|}{0.05^2} (0.8\hat{\imath} + 0.6\hat{\jmath}) = (1.15 \times 10^7 \hat{\imath} + 8.64 \times 10^6 \hat{\jmath})\ N/C$$

or with angle

$$\vec{E}(P) = \left(8.99 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{|4 \times 10^{-6}|}{0.05^2} (\cos 36.8\, \hat{\imath} + \sin 36.8\, \hat{\jmath}) = (1.15 \times 10^7 \hat{\imath} + 8.64 \times 10^6 \hat{\jmath})\ N/C$$

Before you go: are the directions of these vectors what you expected? What would happen if the charge is negative? **Try it!**

QUESTION 6

The force and the electric field are related together by the following equation: $\vec{F} = Q\vec{E}$

a) $\vec{F} = 2 \times 10^{-8} \cdot (4.00 \times 10^7 \hat{\jmath}) = +0.800\, \hat{\jmath}\ N$

b) $\vec{F} = 2 \times 10^{-8} \cdot (-4.00 \times 10^7 \hat{\imath}) = -0.800\, \hat{\imath}\ N$

c) $\vec{F} = -2 \times 10^{-8} \cdot (1.15 \times 10^7 \hat{\imath} + 8.64 \times 10^6 \hat{\jmath}) = (-0.230\hat{\imath} - 0.172\, \hat{\jmath})\ N$

Before you go: are the directions of these vectors what you expected? If the test charge placed in the E-field is positive, what would happen to the direction and magnitude of the force acting on it?

QUESTION 7

a) According to Newton's second law (yes, physics NYA is still valid!!) : $\vec{F} = m\vec{a}$

That is equal to the electric force experience by a charged in an electric field: $\vec{F} = m\vec{a} = Q\vec{E} \rightarrow \vec{a} = \frac{Q\vec{E}}{m}$

$$\vec{a} = \frac{(-1.602 \times 10^{-19} C)(0\hat{\imath} + 2 \times 10^2 \hat{\jmath}) N/C}{9.11 \times 10^{-31} kg} = (0\hat{\imath} - 3.51 \times 10^{13} \hat{\jmath})\ m/s^2$$

Note: as you can see, the force (and thus acceleration) are in opposite direction with the E-field in which it is placed.

b) Remember the projectile motion in physics NYA….

x-direction	**y-direction**
$v_{ox} = 2 \times\ 10^6 \frac{m}{s}$ (constant)	$v_{oy} = 3 \times 10^6 \frac{m}{s}$
$\Delta t = 2 \times 10^{-7} s$	$\Delta t\ =\ 2 \times 10^{-7} s$
$a_x = 0 m/s^2$	$a_y = -3.51 \times 10^{13} m/s^2 \hat{j}$
$\Delta x = ??$	$\Delta y = ??$

Kinematics equations – since the acceleration is constant:

$$\Delta x = v_{ox} t = 0.40\ m$$

$$\Delta y = v_{oy} t + \frac{1}{2} a_y t^2 = -0.103\ m$$

the initial position: $\vec{r}_o = (0, 0) m$
the final position: $\vec{r} = (0.400, -0.103)\ m$

c) The velocity is a vector, so we have to find the x and y components:

In the x-direction:
$v_x = 2 \times 10^6\ m/s$ (constant)

In the y-direction
$v_y = v_{oy} + a_y t = -4.02 \times 10^6 \frac{m}{s}$

The velocity: $\vec{v} = (2.00 \times 10^6 \hat{\imath} - 4.02 \times 10^6 \hat{\jmath})\ m/s$

QUESTION 8

We first have to find the x and y components of the velocity vector:

$$\vec{v} = (6.00 \times 10^6 cos45\ \hat{\imath} + 6.00 \times 10^6 sin45\ \hat{\jmath})\ m/s = (4.24\hat{\imath} + 4.24\hat{\jmath}) \times 10^6\ m/s$$

a) In the y-direction:

$$F_y = m a_y = q E_y \quad \rightarrow \quad a_y = \frac{q E_y}{m}$$

$$a_y = \frac{-1.6 \times 10^{-19} \cdot 2 \times 10^3}{9.11 \times 10^{-31}} = -3.51 \times 10^{14} \hat{\jmath}\ m/s^2$$

$$v_{oy} = 4.24 \times 10^6\ m/s$$
$$\Delta y = 0.02\ m$$
$$\Delta t = ??$$

Solve for *t* from kinematics:

$$\Delta y = v_{oy} t + \frac{1}{2} a_y t^2$$

$$2 \times 10^{-2} = 4.24 \times 10^6 t + \frac{1}{2}(-3.51 \times 10^{14}) t^2$$
$$1.76 \times 10^{14} t^2 - 4.24 \times 10^6 t + 2 \times 10^{-2} = 0$$

Solve using the (famous) quadratic equation:

$$t = \frac{4.24\times10^{6} \pm \sqrt{(4.24\times10^{6})^{2} - 4(1.76\times10^{14})(2\times10^{-2})}}{2(1.76\times10^{14})}$$

$$t = 6.46\times10^{-9}\ s \quad \text{or} \quad t = 1.77\times10^{-8}\ s$$

The time $t = 6.46\times10^{-9}\ s$ corresponds to the time taken for the charge to travel $y = 2\ cm$ on its way up. The time $t = 1.77\times10^{-8}\ s$ corresponds to the time taken for the charge to travel $y = 2\ cm$ on its way down. So it hits the top plate after $\boxed{t = 6.46\times10^{-9}\ s}$.

How far has it moved horizontally in that time?

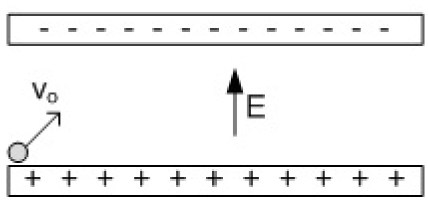
In the x-direction

$v_{oy} = 4.24\times10^{6}\ m/s$ $\qquad$ $a_x = 0$ since $E_x = 0$ $\qquad$ $t = 6.46\times10^{-9}\ s$

$$\Delta x = v_{ox}\Delta t$$

$$4.24\times10^{6}\cdot 6.46\times10^{-9} = \boxed{2.74\times10^{-2}\ \ m}$$

b)



The electron just misses the top plate. It leaves the field at the point [4; 2] cm

x-direction	**y-direction**
$v_{ox} = 4.24\times10^{6}\ m/s$	$v_{oy} = 4.24\times10^{6}\ m/s$
$\Delta x = 0.04\ m$	$\Delta y = 0.02\ m$
$a_x = 0$ and $\Delta t = ?$	$a_y = ?$ and $\Delta t = ?$

$$\Delta x = v_{ox}\Delta t \quad \rightarrow \quad \Delta t = \frac{\Delta x}{v_{ox}} = 9.43\times10^{-9} s$$

$$\Delta y = v_{oy}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad a_y = \frac{\left(2(\Delta y - v_{oy}t)\right)}{t^2} = -4.50\times10^{14}\ \hat{\jmath}\ m/s^2$$

The force acting on the charge is related to the acceleration of the charge and the electric field in which the charge travels.

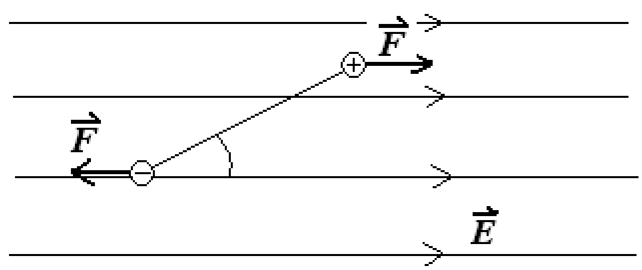
$$\vec{F} = m\vec{a} = Q\vec{E} \rightarrow \vec{E} = \frac{ma_y}{Q}$$

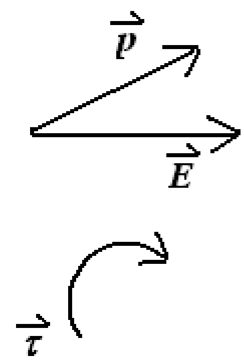
$$\frac{(9.11 \times 10^{-31} kg)\left(-4.5 \times \frac{10^{14} m}{s^2}\right)}{-1.602 \times 10^{-19} C} = 2.56 \times 10^3\, \hat{j}\, \frac{N}{C}$$

$$\vec{E}_{minimum} = (0\hat{\imath} + 2.56 \times 10^3\, \hat{j})\frac{N}{C}$$

QUESTION 9

Visualize the situation:





a) $p = qd = (1e)(4 \times 10^{-10} m) = 4 \times 10^{-10} e \cdot m$

$\vec{p} = 4.00 \times 10^{-10}\ e \cdot m$ at 35°

b)

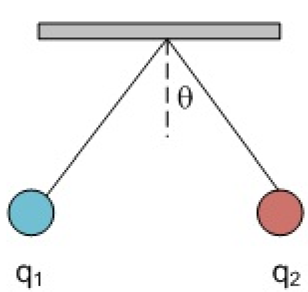
$$\tau = pEsin\theta$$

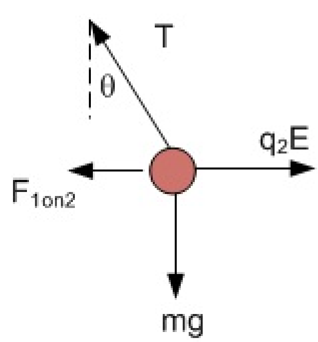
$$\tau = (4 \times 10^{-10} e \cdot m)\left(4.8 \times 10^{-16} \frac{N}{e}\right) sin35 = 1.10 \times 10^{-25} N \cdot m$$

$\tau = 1.10 \times 10^{-25} \hat{k}\ N \cdot m$ (clockwise as defined by the rotation axis above!)

QUESTION 10

You must draw a free body diagram. Both charges will have the same free body diagram. So let's draw it for q_2. Four forces are present: the tension [T], the weight [mg], the electric force between the two charges [F_{1on2}] and, the electric force due to the charge placed in the external electric field [q_2E].





1) $\Sigma F_x = q_2 E - T\sin\theta - F_{12} = 0$

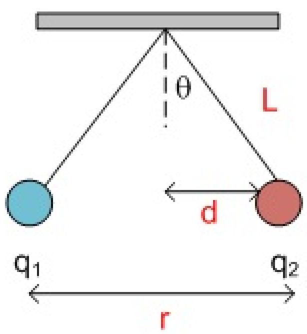
with $F_{12} = \left|\dfrac{k_e q_1 q_2}{r^2}\right|$

2) $\Sigma F_y = T\cos\theta - m_2 g = 0$

$\Rightarrow T = \dfrac{m_2 g}{\cos\theta}$

$$\Sigma F_x = q_2 E - \left[\frac{m_2 g}{\cos\theta}\right]\sin\theta - F_{12} = 0$$
$$= q_2 E - m_2 g\tan\theta - F_{12} = 0$$

The distance r between the charges can be obtained by geometry.



$d = L\sin\theta$

$r = 2d = 3.42\,cm$

Solve for the electric field from the above equation:

$\Sigma F_x = q_2 E - m_2 g\tan\theta - F_{12} = 0$

$$\Rightarrow E = \frac{F_{12} + m_2 g\tan\theta}{q_2}$$
$$= \frac{\left|\dfrac{k_e(5\times10^{-8})^2}{0.0342^2}\right| + 2.00\times10^{-3}\cdot 9.81\tan 10}{5\times10^{-8}}$$
$$= 4.54\times10^5\,N/C$$

QUESTION 11

Lets first find the force that each ion exerts on ion **A**.

Force B on A

$$\vec{F}_{BA} = \left|\frac{k_e e^2}{d^2}\right|(-\hat{j})$$

Force C on A

$$\vec{F}_{CA} = -\vec{F}_{BA} = \left|\frac{k_e e^2}{d^2}\right|\hat{j}$$

Force D on A

$$\vec{F}_{DA} = \left|\frac{k_e e^2}{d^2}\right|\hat{i}$$

Force E on A

$$\vec{F}_{EA} = \left|\frac{k_e e^2}{2d^2}\right|\left[-\frac{d}{d\sqrt{2}}\hat{i} - \frac{d}{d\sqrt{2}}\hat{j}\right]$$

Force F on A

$$\vec{F}_{FA} = \left|\frac{k_e e^2}{2d^2}\right|\left[-\frac{d}{d\sqrt{2}}\hat{i} + \frac{d}{d\sqrt{2}}\hat{j}\right]$$

a) **Net force on A due to B, C, D, E and F**

$$\vec{F}_A = \left|\frac{k_e e^2}{d^2}\right|\hat{i} - \left|\frac{k_e e^2}{2d^2}\right|\left[\frac{2d}{d\sqrt{2}}\hat{i}\right]$$

$$= \left|\frac{k_e e^2}{d^2}\right|\hat{i} - \left|\frac{k_e e^2}{d^2\sqrt{2}}\right|\hat{i} = \left|\frac{k_e e^2}{d^2}\right|\left[1 - \frac{1}{\sqrt{2}}\right]\hat{i}$$

b) The attraction, from the **B, C, D** ions is stronger than the repulsion exerted by the **E, F** ions. The net force is an attractive force. We have only considered the nearest ions because the force is strongest along the shortest distances between the atoms in the crystal. At other distances, the force is slightly smaller: $\vec{F} \propto \frac{1}{r^2}$.

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