Evaluating uncertainty

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I. Estimating uncertainty in measurements

Every measurement has uncertainty. We can never know the true value exactly; we can only estimate it within the limits of our equipment and methods. Because of this, *scientists cannot definitively prove anything*. Instead, science relies on gathering evidence that supports or challenges hypotheses. Over time, this process builds a more reliable understanding of the natural world, where decisions are made taking uncertainties into account. You are expected to reasonably estimate the uncertainties of your measurements in your physics lab (see : Considerations for estimating uncertainty depending on the measuring tool and apparatus in use.Table 1).

Table 1: Considerations for estimating uncertainty depending on the measuring tool and apparatus in use.

Apparatus	Uncertainty
Brass Masses	If using standard masses & standard 50 g hook, assume 0 (it is very small) If you are using the tiny 5 g hooks: ± 1g
Capacitance	Look at markings on the capacitor: • If indicated: use that percentage. • If no indication: ±20%
Capacitance meter	1.5% of reading + 1 last digit
Digital scale (mass)	±1 of the smallest digit displayed
Digital Vernier calipers	±1 of the smallest digit displayed
Dynamometer (spring scales for measuring force)	First, check that the spring is at level zero when there is zero force • if wobbly, e.g., a human is pulling: ± half the interval of fluctuation • if stable: ±1 of the smallest unit on the scale
Human reaction time	200 ms, according to Wikipedia: https://en.wikipedia.org/wiki/Mental_chronometry#Standard_reaction_time_paradigms

Apparatus	Uncertainty
Manual Vernier: calipers, traveling microscope, & spectrometre	Novice users: by how much would two novice users disagree if they measure the same position? Expert users: half of the smallest division (you are not an expert user)
Protractor	What angle does the thickness of the line correspond to?
Spark timer	Assume zero (uncertainty is very small)
Resistance	Determined by the colour of the final band • gold ±5% • silver ±10%
Ruler	Are you measuring a centre-to-centre distance between two points? • ± the average thickness of those points. When measuring a line-of-best-fit you must consider if all the points in your data set are perfectly aligned. If one of the measurement points is a bit off the line, what is the difference between the distance measured along the line of best fit and directly measuring the distance between two adjacent points - this is added to the uncertainty of the points' thickness.
Multimeter	If reading is fluctuating, take the average of the highest and lowest value, and take the uncertainty as half of that interval. If reading is steady, follow manufacturer guidelines: • DC Voltage: 0.5% of reading +1 of the last digit. • DC Current: 0.8% of the reading +1 of the last digit. • Resistance: 0.5% of the reading +1 of the last digit.
Wall thermostat	± 1 °C

Pasco interface and sensors: https://www.pasco.com/products/interfaces-and-data-loggers/850-universal-interface	
Pasco position sensor	Range is 0.15 m to 8 m, depending on sampling rate. Resolution is 1 mm. These are the minimum uncertainties (best-case scenarios): • position: ±1 mm BUT consider what you are measuring • velocity: ± 2 mm/s but better to use uncertainty of x-t line fit • acceleration: ± 3 mm/s^2 but better to use uncertainty of v-t line fit
Capstone magnetic field sensors	± 0.01 gauss or 1 mT
Voltage sensor	±3 mV
Current sensor	±60 mA
Voltage generator	±7 mV

Pasco interface and sensors: https://www.pasco.com/products/interfaces-and-data-loggers/850-universal-interface	
Frequency generator	±1 mHz

II. Uncertainty of a calculated quantity: rules for uncertainty propagation

To find the uncertainty of a calculated quantity, we first must know the uncertainty of each value in the calculation.

- **Constants:** assume an uncertainty of zero.
- **Multiplication/division by exact number**: multiply/divide uncertainty by that exact number.
- **Values calculated from measurements:** propagate the uncertainty of all measured values using Equation 1 (also see Table 2), or using other tools provided by your teacher. Equation 1 is used to calculate the uncertainty of *x*, where *x* is any function of $a \pm \sigma_a$ and $b \pm \sigma_b$.

$$\sigma_{x(a,b)}^2 = \left(\frac{\partial x}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial x}{\partial b}\right)^2 \sigma_b^2 \qquad (Eq.1)$$

Table 2: Use this table to find the appropriate equation for estimating the uncertainty (σ_x) of your calculated value (x)*. They are all derived from Equation 1.*

$x = a + b$ $x = a - b$	$\sigma_x = \sqrt{\sigma_a^2 + \sigma_b^2}$
$x = a \cdot b$ $x = \frac{a}{b}$	$\sigma_x = x\sqrt{\frac{\sigma_a^2}{a^2} + \frac{\sigma_b^2}{b^2}}$
$x = a^n$	$\sigma_x = x \cdot \sqrt{n^2 \cdot \frac{\sigma_a^2}{a^2}} = x \cdot \left\lvert n \cdot \frac{\sigma_a}{a} \right\rvert$
$x = a^n \cdot b^m$	$\sigma_x = x \cdot \sqrt{n^2 \cdot \frac{\sigma_a^2}{a^2} + m^2 \cdot \frac{\sigma_b^2}{b^2}}$

III. Uncertainty of the average of several measurements of the same thing

For a small number of samples, which we will call n, we simply propagate the uncertainty using first the a+b formula from Table 2 followed by dividing by the exact number n, which gives:

$$\sigma_{\bar{x}} = \frac{\sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}}{n} \qquad (Eq.2)$$

For a large enough sample (n > 30), use standard deviation to represent the uncertainty of the average.

$$\sigma_{\bar{x}} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}} \qquad (Eq.3)$$

IV. Uncertainties of Slopes and y-intercepts from applications

Microsoft Excel

Use standard errors as given by LINEST (line statistics) function

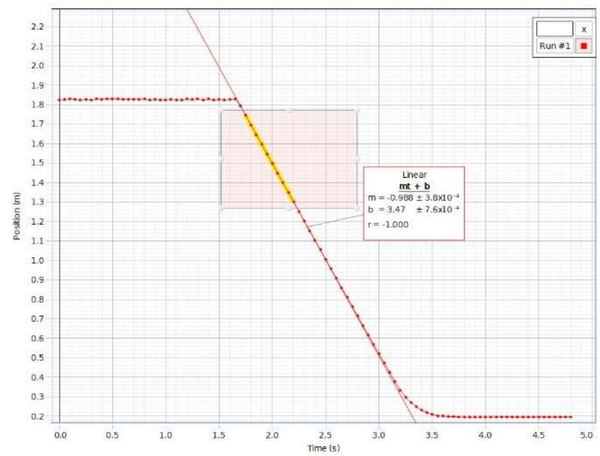
```
=LINEST(known_ys,known_xs,TRUE,TRUE)
```

this function returns 10 values in the following order

1. Slope, m	6. y-intercept, b
2. Standard error for slope, s_m	7. standard error for y-intercept, s_b
3. Coefficient of determination, R^2	8. Standard error for the y-estimate
4. F-statistic	9. Degrees of freedom
5. Regression sum of square	10. Residual sum of square

Capstone

1. Use the highlighter to select part of the graph
2. Use the line fit tool
3. Uncertainties appear next to slope and y-intercept



V. Rounding and number of significant figures

Uncertainty is given to 1 significant figure. The value itself must be rounded to match the precision of the uncertainty. More examples can be found in the Physics Lab Report Guide.

Examples:

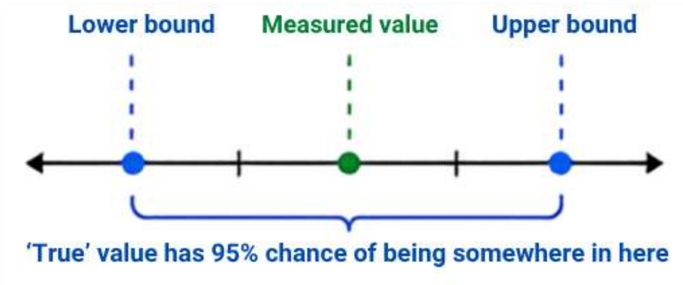
- 12.57 ± 0.03 cm → 12.57 ± 0.03 cm
- 12.57 ± 0.3 cm → 12.6 ± 0.3 cm
- 12.57 ± 3 cm → 13 ± 3 cm

VI. What does my uncertainty tell me?

Uncertainty allows us to make informed decisions about our results. Because all data has uncertainty, we can never know if our experimental results exactly match the predictions of our hypothesis. However, uncertainty allows us to estimate how confident we are that any difference between our data and our predictions is *not* significant. In our physics labs, we will use a 95% confidence interval (CI) to determine whether our data support our hypotheses (i.e., our theoretical models).

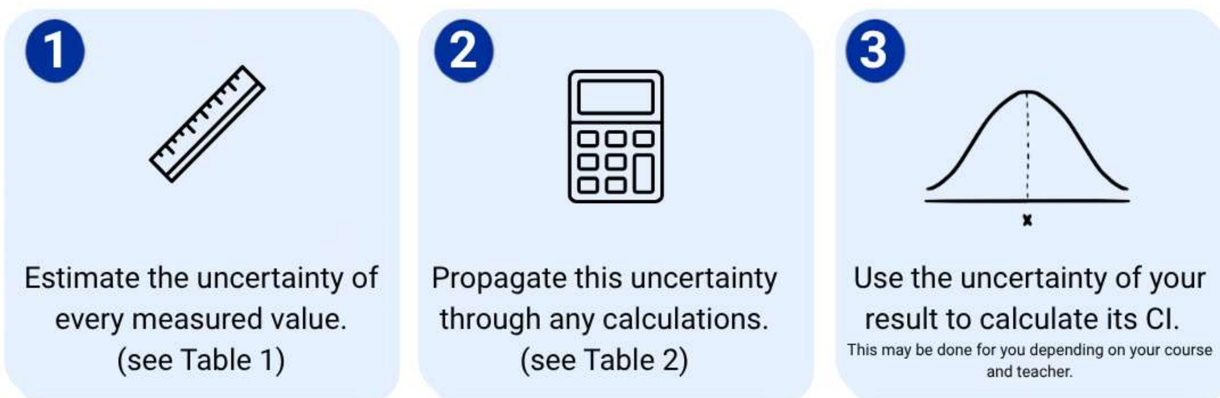
What is a confidence interval?

A 95% confidence interval (CI) is the range of values within which we are 95% confident the true value lies.



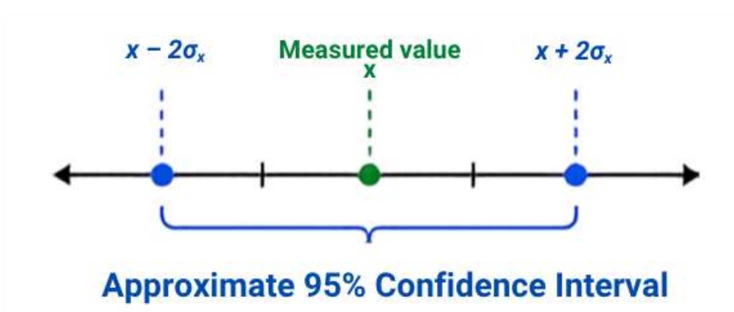
How do I calculate a CI?

Finding a CI for an experimentally determined value requires three steps:



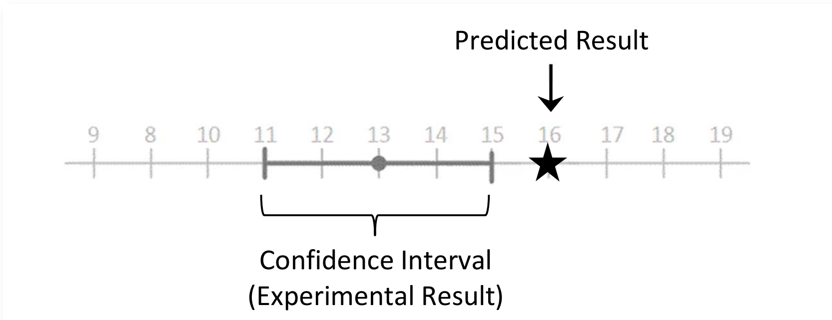
A quick and "dirty" way to approximate the 95% CI is given in Equation 4 below. **Error! Reference source not found.** However, this is only statistically somewhat valid if you have enough data. Only use this method if you know you have enough data or if your teacher tells you to.

$$95\%\ CI\ of\ x = [\underbrace{x - 2\sigma_x}_{\text{lower bound}},\quad \underbrace{x + 2\sigma_x}_{\text{upper bound}}] \qquad (Eq.4)$$

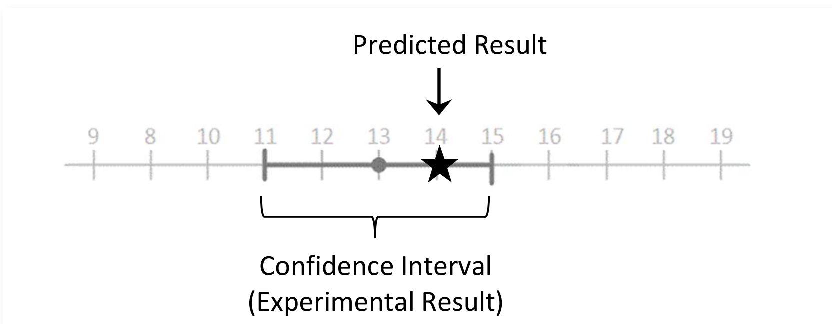


How do I use a confidence interval to make decisions?

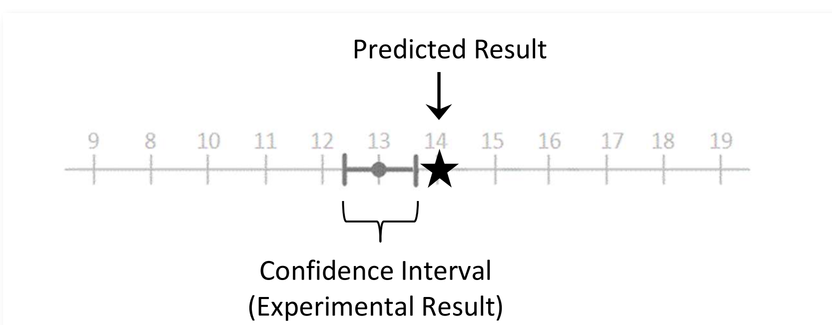
- If the predicted value is outside of our data's CI, we are 95% confident that the prediction is not the same as what we observed experimentally. There may be a problem with our theoretical model (hypothesis).



- If the predicted value is inside our data's CI, we cannot say that the values are different BUT we also cannot claim that they are the same. In such cases, we can say that there is no evidence of a significant difference between the two values.



However, we must consider that a predicted value falling within the confidence interval does not always mean the model is accurate—it may instead be due to the confidence interval being too wide. This issue can be addressed by repeating the experiment with more precise methods to reduce the uncertainty.



The experimental results diagrammed directly above have the same predicted and experimental values as the previous diagram. However, the reduced uncertainty in this experiment reveals a significant difference that was hidden by the large uncertainty of the original experiment.